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A Fundamental Approximation in MATLAB of the Efficiency of an Automotive Differential in Transmitting Rotational Kinetic Energy

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A Fundamental Approximation in MATLAB of the Efficiency of an Automotive Differential in Transmitting Rotational Kinetic Energy

by

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A Fundamental Approximation in MATLAB of the Efficiency of an Automotive Differential in Transmitting Rotational Kinetic Energy

by

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SUPERVISOR: Ronald D. Matthews

The VCOST budgeting tool uses a drive cycle simulator to improve fuel economy predictions for vehicle fleets. This drive cycle simulator needs to predict the efficiency of various components of the vehicle's powertrain including any differentials. Existing differential efficiency models either lack accuracy over the operating conditions considered or require too great an investment. A fundamental model for differential efficiency is a cost-effective solution for predicting the odd behaviors unique to a differential. The differential efficiency model itself combines the torque balance equation and the Navier-Stokes equations with models for gear pair, bearing, and seal efficiencies under a set of appropriate assumptions. Comparison of the model with existing data has shown that observable trends in differential efficiency are reproducible in some cases to within 10% of the accepted efficiency value over a range of torques and speeds that represents the operating conditions of the differential. Though the model is generally an improvement over existing curve fits, the potential exists for further

improvement to the accuracy of the model. When the model performs correctly, it represents an immense savings over collecting data with comparable accuracy.

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Chapter 1: Introduction

Chapter 1 introduces the reader to the purpose of this thesis and the project of which it is a part. The final paragraph of this chapter will alert the reader to the rest of the content of the thesis.

The undertaking of this thesis was to produce a methodology by which the efficiency of a differential or configuration of differentials in transmitting rotational kinetic energy and torque from the transmission output of a vehicle to the wheels can be approximated based on fundamental engineering concepts regarding the studies of tribology, dynamics, fluid mechanics, and heat transfer. This methodology was applied to measurements taken from a selection of automotive differentials to produce a collection; spanning a representative set of powertrain configurations and duty classifications; of generic, fundamental models. Each model has an associated powertrain configuration and duty classification, and each model provides an approximation of the efficiency of the differential or configuration of differentials in any automobile with that powertrain configuration and duty classification.

The approximating models were produced as computer programs in the MATLAB coding language primarily so that the models would coalesce with the rest of the MATLAB functions that provide the ability to predict fuel economy for TxDOT Project 0-5974 for the Texas Department of Transportation.

The Texas Department of Transportation funded TxDOT Project 0-5974 in order to enhance the department's ability to operate and, specifically, to develop a budget for each biennium with reasonable accuracy. TxDOT is still feeling the political ramifications of their billion-dollar budget error from the last decade. However, the budgeting tool from TxDOT Project 0-5974 is able to assist TxDOT in several of its current endeavors, such as predicting fuel tax revenue for the coming years, predicting the total cost of operation of a vehicle fleet, and evaluating the costs and benefits of existing infrastructure and planned construction. These kinds of information will assist TxDOT in budgeting taxpayer money with greater confidence and providing taxpayers with the optimal transportation infrastructure.

Fuel tax revenue is of great interest to TxDOT as they recover from their budgeting error. TxDOT's fuel tax revenue comes from the sale of fuel in Texas. (Texas is unique among the 50 states in this regard.) TxDOT can predict the mileage and driving behavior of people consuming fuel purchased in Texas; however, their previous model for translating mileage into fuel consumption (and thus sales), using the fuel economies of the vehicles, was not sufficiently accurate to engender confidence in fuel tax revenue predictions within a billion dollars (as history has shown). Part of the loss of accuracy came from inaccuracies in the prediction of fuel economy. Automobile manufacturers have in the past reported two fuel economies, one for city driving and the other for highway driving. Highway driving is always more fuel efficient and has approximately on the order of 10% greater fuel economy than city driving fuel economy. These two numbers represent data taken from two different driving cycles, the city driving cycle and the highway driving cycle. Despite efforts to capture the driving behaviors of the majority of the population, these driving cycles cannot account for issues like traffic congestion. The tool generated by TxDOT Project 0-5974 allows the user to use any driving cycle that he or she can create in order to better approximate the fuel economy of vehicles in Texas.

The choice of driving cycle is not the only influencing factor on the accuracy of the prediction of fuel economy. Other influencing factors require a deeper fundamental understanding of fuel economy. Fuel economy represents the distance a specified quantity of fuel will allow the vehicle to travel. Road load forces seek to bring the vehicle to a stop, to prevent it from traveling that distance. As a result, a quantity of fuel which contains adequate energy must be provided to the vehicle in order for it to travel. The reader should refer to Chapter 3 regarding fundamental theory for a better understanding of the relationship between work, energy, forces, and displacements. In addition to road load forces, frictional and fluid shearing parasitic losses occur in the machine that harnesses the energy from the fuel. These losses prevent the vehicle from using some of the energy liberated from the fuel to overcome these road load forces over the desired traveling distance. That is, a portion of the fuel’s energy, instead of being converted to rectilinear and rotational kinetic energies, is converted to heat and lost to the air through which the vehicle travels. This is not to be confused with the aerodynamic drag, a road load force associated with forcing the bluff body that is the vehicle through the air above the road. By focusing on the losses between the energy liberated from the fuel and the energy that moves the vehicle from one location to another, a quantity can be defined that is the efficiency of the powertrain, that machine that harnesses energy from fuel and provides energy to the vehicle to allow it to travel some distance.

The efficiency of the powertrain can be shown to be the product of all of the efficiencies of its components. In turn, these efficiencies are related to the fuel economy that the vehicle produces. For instance, imagine a powertrain that is 50% efficient. In order to deliver a desired torque to the tire(s) while traveling at a given speed, twice the "power" (chemical energy addition rate) that is delivered to the tire(s) must be provided to the powertrain. Therefore, twice the fuel must be provided to the engine than would be necessary if the powertrain were 100% efficient. Utilizing twice the fuel causes the fuel economy to be cut in half. In general, fuel economy is directly proportional to powertrain efficiency, and both are inversely proportional to fuel consumption. As a result, it is very important to accurately quantify the efficiencies of the components of the powertrain in order to ensure the accuracy of a fuel economy prediction. Accurate fuel economy predictions allow for an accurate estimate of fuel consumption, which can be used to forecast future fuel tax revenue or to evaluate infrastructure maintenance and new construction.

Other programs exist that can be adapted to approximating the fuel economy of a fleet of vehicles. Some of these programs include ADVISOR, ADAMSCAR, Autonomie, CONVERGE, PSAT, and WAVE.

The budgeting program produced in TxDOT Project 0-5974 is more flexible than these other programs in that it both produces values of fuel economy and applies these values to a wide variety of budgeting scenarios by simply navigating menus.

It was shown that the efficiency of the powertrain and the efficiencies of all of its components are important quantities for calculating fuel economy. Because automotive differentials exist between the engine and the tires of all currently available four-wheeled vehicles, differentials are part of the powertrain. Thus, the efficiency of a differential or configuration of differentials is of interest in predicting fuel economy.

Two easier methods for approximating the efficiency of a differential exist, but neither is a fundamental model. First, data can be collected for the efficiency of a "typical" differential of a vehicle operating at an average speed over average terrain. The efficiency of a differential can then be assumed to be the average efficiency of the data collected. This method will fail to capture four of the five investigated factors that affect the efficiency of the differential: torque, speed, gear ratio, and duty classification. However, an intelligent model utilizing this efficiency value could conceivably still account for powertrain configuration. It is suggested that the reader refer to the section on powertrain configurations (in Chapter 4) for a deeper understanding of this.

Another method for approximating the efficiency of a differential is to take data for every conceivable value of any or all of the effects mentioned in the preceding paragraph (and more). The quantity of data required to develop a five-dimensional efficiency map with an appreciable amount of resolution is reasonable for the average computer memory storage device. The product of powertrain configurations and duty classifications will produce the number of vehicles for which data would be collected. Effects of speed and torque would require a resolution of 10 speed values by 100 torque values easily. Finally, the effects of gear ratio would likely require a resolution of at least 10 distinct gear ratios. Assuming that 1000 models of vehicle are currently in operation, this method would require the collection (on a chassis dynamometer, using sophisticated strain gauges, and having completed coastdown tests) of efficiency values numbering at least ten million (10^7). The resolution for the efficiency should allow the ability to discern between 1000 values. This can be accomplished by using 10 binary terms per value, or roughly one byte of data per value. This would necessitate the ability to store 10 MB. A computer's operating system today is usually 100 times that size, and computer users are typically allowed 10-100 times as much storage as the operating system. Computers have been capable of storing such an efficiency map for the last decade or two, according to the rule that storage density has doubled every year.

The main problem with completing a data set lies in the cost of testing. First, the chassis dynamometer and strain gauges need to be purchased, as does the test bed in which they are to be installed. Second, coastdown tests need to be completed in order for the chassis dynamometer to simulate road load conditions. Third, the equipment must be attended, most feasibly by humans, while the data collection process occurs. Finally, the time required to logistically use vehicles and then store them or return them to their owners would present a challenge.

A third option is to model the efficiency of the differential using fundamental engineering principles. This is the least expensive option and offers the interested engineer the greatest flexibility. The difference between the computer model and the first, easier method is that the effects of the five parameters mentioned are not simply discarded but are considered in the computer model. In order to account for effects of torque and speed within a duty classification, a research team member on this project, PhD candidate Kyung Jin Kim, a co-investogator on TxDOT Project 0-5974, suggested a method for comparing distinct differentials with the same gear ratio but different load capacities. In this respect, the automotive differential out of one front-wheel-drive automobile can be compared with the automotive differential out of another front-wheel-drive automobile in the same duty class, provided the gear ratios are the same or similar. Furthermore, this model provides a method for estimating the effect of gear ratio on the construction of the differential based on an understanding of the modes of failure of differentials. All of this capability is available within the computer model, and proof-of-concept (POC) tests have validated the model. The reader should check Chapter 7 for the POC tests.

A basic understanding and key concepts of automobile powertrains is provided in Chapter 2. Chapter 3 is a general discussion regarding the fundamental engineering concepts that apply to automobile drivetrains. Chapter 4 provides a description of the eight configuration/duty classification combinations of differentials within automobile drivetrains that were chosen to represent the entirety of the vehicle market. The modeling and programming methodologies for creating the associated MATLAB programs are discussed in Chapter 5. Chapter 6 discusses three different potential modes of operation for the programs. Proof of concept tests that were performed to validate the models are discussed in Chapter 7. Chapter 8 provides a summary of the results of the proof of concept tests and discusses the conclusions regarding the validity of the models. Chapter 9 provides the academic community with recommendations for future work.

Chapter 2: Automotive Basics

This chapter will connect the concept of fuel economy with the fundamental engineering concept of efficiency. The chapter will also discuss the common architecture of automotive powertrains as well as some of their differences within duty classification and configuration of components.

The introduction mentioned that people that drive in Texas buy gas and pay both a state fuel tax and a federal fuel tax. TxDOT receives all of the state fuel tax and much of the federal fuel tax to maintain state and federal roadways and build new roads. This construction and maintenance ultimately serves to improve the lives of these drivers not only by saving them time on their commutes but also by improving the fuel economy of their vehicles. The introduction discussed the relationship between fuel economy and the consumption (and purchase) of fuel. Fuel economy can only be determined empirically; however, the concept of powertrain efficiency traces back to fundamental principles. These fundamental principles are discussed in Chapter 3. However, the powertrain, its components, and its interaction with the vehicle and imposed road load forces are the subjects of the present chapter.

The most common automotive powertrain starts with some combination of a combustion engine and/or an electric motor and also includes a transmission and at least one differential. Electric vehicles and hybrids deserve mention because they have been around as long as combustion engine vehicles. Consumers must still pay for the electrical energy required to charge their vehicles, so this electrical energy can be and generally is thought of as fuel for electric and hybrid vehicles. For the sake of convenience, both electric motors and combustion engines serve to liberate chemical energy (from batteries or refined petroleum products) and convert the energy's form (by electromagnets or by pneumatic expansion and piston-cylinder assemblies) into rotational kinetic energy. In this regard, an automotive powertrain can be thought of as containing components including an energy source and converter, a transmission, and at least one differential. As previously speculated, each of these components will transmit some of the energy provided to them by upstream components but will also divert some portion of that energy to heating the surroundings because of the phenomena of friction and fluid shear.

Before discussing motors and engines further, the functions of transmissions and differentials will be introduced to allow for a complete picture of a functional powertrain. The main purpose of the transmission is to keep the engine or motor turning at as close to its optimal rotating speed as possible for any vehicle speed. The optimal rotating speed of each engine or motor is unique to its design. Combustion engines typically produce their peak torque at rotational speeds near the speed at which they are the most efficient, which provides a convenient target. It is not realistic, however, to expect an engine to be able to both provide enough torque to the powertrain at very low speeds to accelerate while also providing optimal torque to the powertrain at high speeds and allowing the vehicle to continue to accelerate. The transmission controls the rotational speed ratio between the engine and the wheels.

An automobile differential is an application of epicyclic gearing to address the problem of simultaneously powering two axles while allowing the axles to spin independently of one another. Before the introduction of differentials, an automobile transmission would be linked by gears or by a chain/sprocket drive or some other means to one wheel or one or both locked axles of the automobile. This difference in the number of wheels being powered represents an intrinsic trade-off. As the number of powered wheels increases, so does the maximum force that can be developed at the tire-road interface before the wheels begin to slip. The tractive limit of any tire is based on the weight placed upon it, ignoring differences in the coefficient of friction. Hence, adding more wheels is like adding more weight for the friction limit. This, in turn, allows the driver to take full advantage of the acceleration ability of the vehicle. Cornering (and likely deceleration through braking), by contrast, is handled by all of the wheels, although dominated by the outside tires (and the outside front tire if cornering while braking). However, if the powered wheels are not able to turn independently of one another, a non-ideal condition may cause one or more of the tires to slip on the road surface to accommodate this condition. A bump, for instance, would require the bumping wheel(s) to travel faster than the other wheels.

During a turn, as well, the wheels on the outside of the turn must travel faster and thus rotate faster) than their counterparts on the inside of the turn (unless the back is being slung outward from the tracks from the inner wheels, in which case the wheels in back travel faster than the front wheels). This phenomenon is easily observable by driving a vehicle in one complete circle on a dirt road. Four separate circles will be observed, all having a common center but each having a different radius. For each wheel to travel the circumference of its circle in the same time it takes to sweep through the entire circle, some wheels must travel faster or slower than others. Before the application of epicyclic gearing to the powertrain (i.e. the differential), axles forced to sweep in circles would have a tire digging into the road and bouncing around. That is, before the invention of the differential, cornering was unpleasant at best.

While the tires slip at the tire-road interface, the available coefficient of friction drops slightly from the static to the kinetic value. Furthermore, an initial study of tribology will elicit an understanding that the rubbing motion will produce undue wear in the tire, decreasing its life. Hence, as the number of tires that were driven increased, the worse the wear on the tires became and the more frequently tires would need to be repaired or replaced. As the number of tires that were driven decreased, the acceleration capability of a vehicle decreased near-proportionally. Performance was pitted against cost, both of which are driving factors for the consumer. The differential allows two (or more, for more complicated configurations) wheels to be operated at different speeds while simultaneously powered.

For all of the benefits that engines, motors, transmissions, and differentials bestow on powertrains, their level of sophistication by necessity dictates an inferior performance to simpler designs at the design conditions. For instance, if a transmission could be instantaneously replaced with a single gear pair with the same ratio that the transmission is using, less potential to use the energy to overcome road load forces would be lost in the contact and lubrication of the gears than for all of the gears of a full transmission. For the consideration of efficiency and for an overall understanding of how fuel economy is derived, engines and motors are of the utmost interest; in terms of efficiency calculations, transmissions and differentials are secondary devices in comparison with the power source of a vehicle.

As previously alluded to, for internal combustion engines, the energy to overcome the resistance from road load forces along the path of travel comes from refined petrochemical fuel. The product of the efficiency of the powertrain, the mass of fuel consumed, and the heating value of the fuel provides the amount of energy that was required to traverse the travelled distance against road load forces. Just the product of the mass of fuel consumed and the heating value of the fuel provides the amount of energy that should have been liberated from the fuel. Some of the difference between these energies, as has been previously mentioned, changes forms from rotational and translational kinetic energy to heat through the phenomena of friction and fluid viscosity before the energy can be delivered to the wheels, chassis, and the resistive elements that form road load forces. However, the engine does not extract all of the energy stored in, or even liberated from, the fuel.

Internal combustion engines have four efficiencies associated with their torque production. The combustion efficiency tracks how close to equilibrium the series of chemical reactions of the fuel combusting came and, therefore, what ratio of the chemical energy that could have been liberated from the fuel was, in fact, liberated from the fuel and converted to thermal energy. The indicated thermal efficiency tracks how much of the released thermal energy was converted to useful work at the top of the piston throughout the cycle before the exhaust left the cylinder. The indicated thermal efficiency is always much less than one, as the Second Law of Thermodynamics requires that heat losses occur in a heat engine when the cold reservoir is above absolute zero. The volumetric "efficiency" keeps track of how close the engine got to filling the combustion chamber completely with air. The last efficiency is in quotations because it is not a true efficiency: though some loss of pressure occurs when the air travels from the intake, through the filter, around the intake valve, and into the cylinder, these pressure losses can be offset (and actually overcome) by tuning the intake acoustics. It is possible to have a volumetric efficiency that is greater than 100%. None of the other mechanical components has any efficiency associated with it besides the mechanical efficiency, which is the efficiency of converting the useful work at the top of the piston(s) to useful rotational mechanical energy at the engine's output shaft. Hence, the engine's torque production can be proven to be the product of the four aforementioned fundamental engine efficiencies.

Concern over the operating efficiency of electric motors is beyond the scope of this thesis. It is presumed that inefficiencies exist in extracting the chemical potential energy from the batteries and converting it into rotational and translational kinetic energies. It is sufficient to note that both electric motors and internal combustion engines contain inherent inefficiencies for the purposes of studying transmissions and, more importantly for this thesis, differentials.

Transmissions can vary widely. Their design is based on several factors that are illuminated by the designs. The consumer can choose between manual and automatic transmissions. The main difference in the construction of an automatic transmission over a manual transmission is that an automatic transmission has a torque converter. The torque converter in an automatic transmission replaces the clutch of a manual transmission. From a driving standpoint, eliminating the clutch reduces the burden on the driver, which is further alleviated by computer-based shifting. However, the torque converter of an automatic transmission is inefficient compared to a direct clutch in manual transmissions. Another choice the consumer must make is the number of gears available. As the number of gears available increases, the car has the capacity to perform better. However, each gear represents an extra inefficiency. In order to offset the inefficiency of having a torque converter, the manufacturer of an automatic transmission may include fewer gear ratios to reduce the number of gears. The engineering tradeoff is passed along to the consumer. The innovations don't stop there, however. Some transmission manufacturers have begun to use planetary gear sets, which can provide several gear sets in a very axially compact arrangement. Several such configurations already exist.

It is perhaps less likely that the reader knows how a differential works or what one is than for an engine or transmission. This discussion shall begin with its purpose. The engine supplies the torque, and the transmission allows the driver to use that torque over a wide range of driving speeds. However, in order for the vehicle to turn a corner or drive over a bump, the wheels need to be able to turn at different speeds. Otherwise, the tires will be forced to slip along the road surface. Gravel and dirt roads will allow the tires to slip fairly easily, while stickier surfaces like asphalt will load the tires more heavily. This is why 4-wheel-drive vehicles must only be operated in 4-wheel-drive (4WD) mode only on gravel and dirt roads: instead of having a differential between the front and rear axles, like in an all-wheel-drive vehicle, a 4WD vehicle has a transfer case, which will not allow the front and rear differentials to turn at different speeds. Internal combustion engines and transmissions are older than differentials, but consumers have been taking advantage of differentials for at least the better part of a century.

In order for differentials to allow wheels to turn at different speeds while still powering them, they need some form of epicyclic gearing. Several designs exist, including the conventional design with spider gears and the three types of Torsen (torque-sensing) differentials. They all use some configuration of gears in some form of epicyclic gearing arrangement. Like with transmissions, these gear pairs have an associated mechanical efficiency arising from the phenomena of friction and fluid viscosity.

As previously mentioned, all of these efficiencies are of interest in determining the fuel economy of vehicles when faced with the task of overcoming road load forces. According to Matthews (2010), though the consideration of road load forces neglects resistive loads from wind speed, aerodynamic downforce, and road grade (as in angle of inclination), the other factors that are considered are aerodynamic drag on the chassis and rolling resistance at the tire-road interface. Matthews explains that there is a coefficient of friction between the tire and the road, even if the tires are rolling, though that coefficient of friction is distinct from that of sliding tires. That is the source of rolling resistance in the consideration of road load forces.

While road load force calculations are consistent for all vehicles, though the coefficients will differ, the architecture of the automotive powertrain will differ substantially based on purpose. Each distinct configuration represents the culmination of an engineering tradeoff. Some configurations span multiple duty classifications, while other configurations function optimally only in a subset of the duty classifications. Eight models for the efficiency of a differential or set of differentials were chosen to represent the entirety of the vehicle market. These models span the duty classifications and powertrain configurations discussed in the following subsection.

Section 2.1: Duty Classifications

According to Matthews (2011), the duty classification of a vehicle is based on its Gross Vehicle Weight Rating and/or its Loaded Vehicle Weight. As the weights increase, the duty classification will go from light-duty to medium-duty to heavy-duty. It is worth noting that EPA does not have a medium-duty vehicle classification except for medium-duty passenger vehicles. Although EPA has only this one specific type of medium-duty vehicle, truck classification schemes often include a medium-duty category.

Section 2.1.1: Light-Duty

Light-duty vehicles span Gross Vehicle Weight Ratings from 0-8500 lb. Light-duty vehicles are intended primarily for the transportation of people rather than cargo. Since the weight is sufficiently bounded that the amount of gasoline consumed is not a strong function of vehicle weight, fuel economy is generally computed in a way that ignores vehicle weight, other than indirectly through the coastdown coefficients. Coastdown coefficients are taken for a standard payload meant to represent the driver. Adding more passengers and/or luggage, since light-duty vehicles are not built to transport loads that are much more massive than the vehicle itself, would not have as pronounced an effect on fuel economy than, say, for a heavy-duty vehicle.

Section 2.1.2: Medium/Heavy-Duty

Heavy-duty vehicles have a Gross Vehicle Weight Rating floor, although the value has changed historically. These vehicles also typically have a large frontal area and are generally used to haul cargo instead of people. Weight becomes a significant factor in determining the cost of fuel associated with truck transportation. As a result, fuel economy for heavy-duty vehicles is computed in a way that accounts for vehicle weight. The present differential efficiency model treats medium-duty trucks as heavy-duty vehicles with the appropriate scaling.

Section 2.2: Powertrain Configurations

As previously mentioned, six powertrain configurations were chosen which spanned the duty classifications. In particular, the rear wheel drive (RWD) configuration is shared between the light- and heavy-duty classifications. The dual differential is only available for the heavy-duty classification, and the other configurations (front wheel drive, FWD; four wheel drive, 4WD; and all wheel drive, AWD) are exclusive to the light-duty classification.

Section 2.2.1: FWD

Front wheel drive (FWD) vehicles have a transaxle in which the transmission and differential share the same housing. This provides the differential with the benefit of being exposed to a pressurized lubricant source, the oil pump, which allows less of the differential to be immersed in the more viscous transaxle lubricant, instead experiencing reduced windage losses from the less viscous air inside the transaxle. The engine and transaxle are typically mounted laterally across the front axle, allowing the differential to use a helical gear pair rather than a hypoid gear pair. The helical gear pair is more efficient than the hypoid gear pair.

Section 2.2.2: RWD

Rear wheel drive (RWD) vehicles have a differential on the rear axle. In a rear wheel drive vehicle, a long shaft must traverse the distance between the transmission and the differential. Furthermore, since that shaft is not parallel to the rear axle, the differential cannot use a helical gear pair but must use a hypoid gear pair instead.

Section 2.2.3: 4WD With Transfer Case

Four wheel drive (4WD) vehicles can operate in two modes. In the two wheel drive mode, the transfer case is disengaged, and the vehicle behaves like a RWD vehicle. However, in four wheel drive mode, the transfer case engages the front differential and makes it rotate exactly as fast as the rear differential. The nature of the transfer case does not allow for operation of four wheel drive on pavement which is hilly or curvy, as either one will cause axle bind due to the road wanting to force one of the axles to spin faster than the other. The transfer case is typically a 1:1 chain-drive system with some sort of clutch.

Section 2.2.4: AWD

All wheel drive (AWD) vehicles address the problem of 4WD transfer cases not allowing 4WD operation on pavement. The transfer case is replaced by a center "differential" which may be anything from a conventional differential to simple loose clutch plates. When the center differential is a true conventional differential, it is similar in nature to the FWD differential, although there is no gear pair. In fact, some AWD vehicles use a transaxle to combine the transmission and center differential. The other differentials are hypoid gear differentials, similar to the RWD and 4WD vehicles.

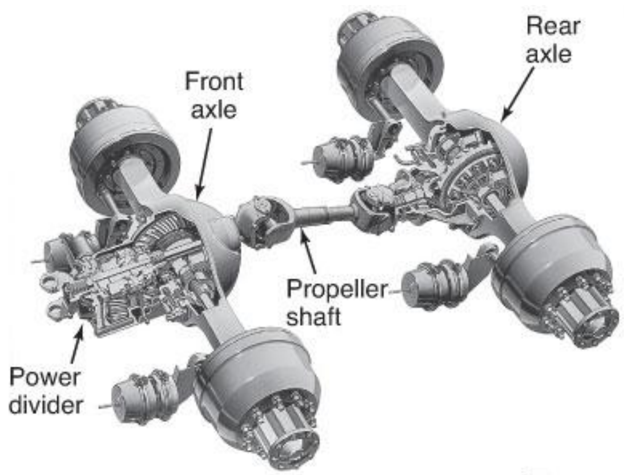


Figure 2.1. A heavy-duty tandem differential (Bennett and Norman, 2011).

Section 2.2.5: Heavy-Duty Differentials

A vast range of vehicles are categorized as heavy-duty: all vehicles with a Gross Vehicle Weight Rating of more than 8500 lb. Thus, many heavy-duty trucks have a single rear axle and a differential that differs from a light-duty rear wheel drive differential only in scale. Large heavy-duty trucks have two rear axles. In this case, they generally use a “tandem” differential, as illustrated in Figure 2.1. In this case, torque is delivered to both rear axles, with the torque split controlled by a “power divider”. However, some heavy-duty trucks have “tag axles”, in which case there is a differential on only the front-most of the two rear axles. Both categories of heavy heavy-duty trucks, those with tandem axles and those with tag axles, can have a two-speed gear box in front of the first input, as discussed in the following subsection.

Section 2.2.6: Dual Differentials

The dual differential is a conventional heavy-duty differential with a two-speed gearbox located directly upstream and sharing the same housing. Though most gearboxes like this are used in multi-stage transmissions, some heavy duty vehicles use a dual differential instead. Dual differentials are certainly a newer addition to differential technology. Having two available axle ratios allows the driver to transport the payload in low gear (with substantially greater torque available to accommodate the increased mass) and return, empty, in high gear (sacrificing torque availability, when it is not needed, for increased fuel economy).

Section 2.2.6.A: Low Gear Ratio

The two-speed gearbox has a low gear and a high gear. The high gear ratio is typically assumed to be unity (1.0:1). However, the low gear ratio is a required input for the model of a dual differential just like all of the transmission gear ratios are required inputs for the transmission model. The need for a low gear ratio is unique to the dual differential.

Section 2.2.6.B: Gear Selection

The two-speed gearbox has the option of in which gear to operate. This shifting is controlled within the fuel economy drive cycle driver model. As a result, unlike the low gear ratio, the user has no control over the gear selection of the dual differential.

Section 2.2.7: Tandem Axles

Automobiles need at least three wheels to be stable, in much the same way that a stool needs at least three legs. Two notable exceptions are motorcycles and trailers. Motorcycles require that the driver constantly balance the vehicle or risk falling over. Trailers, on the other hand, have a towing hitch that acts as a "third leg," so that the trailer doesn't tip forward or backward on its axle. Of course, monowheels exist but have only a small following and are capable of only 50 mph, according to Watkins (2010). There must be a reason that monowheels have earned a place (or two, counting the Purves's Dynasphere) in the book entitled The World's Worst Inventions: The Most Stupid Gadgets and Machines Ever Made. A sidecar motorcycle has exactly three wheels: its driver is not required to constantly balance the vehicle. However, most cars, trucks, and SUV's have four wheels for the same reason that most stools have four legs: further increased stability.

Adding more than four wheels doesn't noticeably increase stability. However, some vehicles have six wheels, either by putting more wheels on the rear axle or by adding another rear axle. The extra wheels are required for hauling heavy loads, as each tire has a maximum load capacity rating. Driving a rear axle with four wheels on it is fairly straightforward; however, driving two rear axles, known as tandem axles, allows the manufacturer to further specialize the powertrain. A live tandem configuration uses a center differential and extra gearing to power differentials, whether conventional or dual, on both rear axles. By contrast, a tag tandem configuration only powers one axle's differential, allowing the other rear axle, known as the "tag axle," to spin freely. The power flow through the tag tandem configuration, when compared with the live tandem configuration, travels through fewer components on its way to the tire-road interface. With fewer sources of loss, one would expect the tag tandem configurations to be more efficient than comparable live tandem configurations. An executive report from the North American Council for Freight Efficiency (NACFE) in November of 2010 quotes "potential fuel economy increases ranging from 2.5 to as much as 6%" from using tag tandems in favor of live tandems. A representative of the NACFE stated that the organization collected this information from a report by the company TIAX and a paper by the National Academy of Sciences (NAS), which cites the TIAX report. The report by TIAX claims a 1% improvement in fuel economy. The NACFE quote may be referring to several technologies simultaneously.

Chapter 3: Fundamental Theory

This chapter will discuss the fundamental engineering principles used in forming the models for the efficiency of an automotive differential. In addition, the assumptions used in forming these models will also be discussed in this chapter.

The fundamental theory behind the model of a generic differential lies within the studies of mechanics, dynamics, heat transfer, and thermodynamics. The governing principles for the model are Newton's Second and Third Laws of Motion and the First Law of Thermodynamics. Newton's Second Law of Motion (e.g., Hibbeler, 2006), states that "a particle acted upon by an unbalanced force **F** experiences an acceleration **a** that has the same direction as the force and a magnitude that is directly proportional to the force." The reader will be more familiar with the equation representation of Newton's Second Law of Motion.

$$\sum_{\text{direction x}} \vec{F}_x = m\vec{a}_x$$

Equation 3.1: Newton's Second Law of Motion

This understanding is necessary to develop a relationship between the moment (or torque) of a force and the angular momentum of the system. Hibbeler writes on page 257 that the sum of moments on a system about a point is equal to the time rate of change of linear momentum, which he then decomposes into two components. If it is assumed that all of the shafts and other pieces of a differential experience negligible deflection with respect to their axes of rotation, then one of the components of angular momentum becomes negligible. The resulting equation is presented on page 385 of Hibbeler's text:

$$\sum_{\text{axis z}} \vec{M} = \sum \vec{M}_z = I\vec{\alpha}_z$$

Equation 3.2: The Moment Balance in Terms of Rotational Moment of Inertia and Rotational Acceleration where I is the rotational inertia and $\vec{\alpha}_z$ is the angular acceleration. This rotational version of Newton's Second Law of Motion is more useful for modeling a differential. The present model for the efficiency of a generic differential will presume operation at steady speed, neglecting inertial effects. The drive cycle simulator that pairs with this model estimates the inertial contribution separately over finite time steps. Use of sufficiently small time steps allows both models to maintain a fair degree of accuracy. These inertial effects are the right hand sides of Equations 3.1 and 3.2: both translational and rotational acceleration are presumed to be zero.

$$\vec{v}_x = \text{const.},\ \therefore \vec{a}_x = 0\ \therefore \sum \vec{F}_x = 0$$

Equation 3.3: Result of Force Balance by Imposing Constraints on Linear Acceleration

$$\vec{\omega}_z = \text{const.},\ \therefore \vec{\alpha}_z = 0\ \therefore \sum \vec{M}_z = 0$$

Equation 3.4: Result of Moment Balance by Imposing Constraints on Rotational Acceleration

Forces, masses, and accelerations are useful for kinetics and kinematics. However, when discussing the use of fuel to propel an object, the reader must turn to the First Law of Thermodynamics, the conservation of energy. Vehicle designers are charged with the task of extracting chemical potential energy from fuel and converting that energy into work to propel the vehicle against the road load. Work is a special form of energy, such as translational or rotational kinetic energy or gravitational potential energy, that can be used directly to accomplish many tasks that are useful to humanity. While work can be converted into thermal energy, not all thermal energy can feasibly be converted into work, according to the Second Law of Thermodynamics. Quantities like forces and moments can be used to calculate energy and work. For instance, the road load energy requirement E_{rl} associated with traversing a given distance d is simply the product of d with the sum of the forces that are resisting motion over that distance (for “road load” conditions – operation over a level road with no wind – the only resistive forces are aerodynamic drag, F_D, and rolling resistance, F_R). As the force one entity exerts on another acts on the other entity while it travels a given distance, energy is transferred between the entities. Similarly, the product of a torque applied on a body and the angle it has turned normal to the applied torque represents the energy that has been transferred between the body and the source of the torque. Some entities can take translational kinetic energy from the car or rotational kinetic energy from the powertrain, while other entities may provide translational kinetic energy to the car or rotational kinetic energy to the powertrain. The fuel, for instance, is used to provide the powertrain with rotational kinetic energy and the entire vehicle, including the powertrain, with translational kinetic energy. The sources of road load, aerodynamic drag and rolling resistance, extract these same energies from the powertrain and the vehicle.

The quantity of energy tells the reader how much fuel will be needed to complete the journey but not whether or not the vehicle will be able to extract a sufficient quantity of the energy in a timely manner to make the journey. Power is the time rate of change of energy, and, like energy, must be conserved. The reader can simply take the time derivative of the First Law of Thermodynamics to verify this assertion. Alternatively, the First Law of Thermodynamics could be seen as the integral in time of an equation that would be more useful in the study of transfer of energies, such as heat. Each of the components of a powertrain has a mechanical efficiency, which is the ratio of power transmitted to the component downstream to power received from the component upstream. Mechanical power, p, is the product of torque, τ, and angular speed, ω.

$$\frac{dW}{dt}=p=\tau*\omega=F*v$$

Equation 3.5: Definition of Power in Terms of Work or Other Useful Parameters

While energy, and therefore power, must be conserved, torque (rotational work) does not have to be conserved. For instance, a gear pair may multiply or divide the torque. However, to conserve power, the speed must be adjusted accordingly, even in the absence of frictional power sinks. It is useful to recall that power is simply the amount of energy, E, which is extracted, transmitted, or delivered over a given period of time, t:

$$p=\frac{dE}{dt}$$

Equation 3.6: Definition of Power in Terms of Energy

It is perhaps easier to think of power as an energy rate, in that respect. In this respect, also, an efficiency is just as well a ratio of energies as it is a ratio of powers.

$$\eta=\frac{p_{out}}{p_{in}}=\frac{E_{out}}{E_{in}}$$

Equation 3.7: Definition of Efficiency in Terms of Energy and Power Ratios

As discussed in the introduction, the efficiencies of the components are of interest in determining the fuel economy of a vehicle. The challenge is in predicting the efficiency of each component of a differential and bringing the whole differential together. As each component is encountered, it will apply a resistive torque on the powertrain. Assuming that sufficient power is supplied from the engine, this resistive torque creates an inefficiency in delivering that torque. As previously mentioned, power is related to torque and angular velocity. Hence, the efficiency of each component can be decomposed.

$$\eta_{component}=\frac{\tau_{out}*\omega_{out}}{\tau_{in}*\omega_{in}}=\frac{\tau_{in}*\omega_{in}-\tau_{component}*\omega_{component}}{\tau_{in}*\omega_{in}}=1-\frac{\tau_{component}*\omega_{component}}{\tau_{in}*\omega_{in}}$$

Equation 3.8: Generic Decomposition of Component Efficiency into Torques and Angular Speeds where $\eta_{component}$ is the torque loss in the component, due to friction, "windage", etc. Other than the gear pair, the speeds will be the same. However, it should be noted that the efficiency of a component has an effect on the torque that is transmitted downstream to the next component.

Section 3.1: Assumptions

Under ideal conditions, everything about the differential's operation would be known every time the model runs. However, too many variables exist for the typical user to know and, thereby, be able to input. Thus, in the present model, some of these variables have been held constant. Other variables have been expressed in terms of the quantities that the user can be expected to provide.

Section 3.1.1: Symmetries

The assumption of symmetry allows the model to simultaneously solve for the two or more outputs from a power splitting device. After all, the differential's purpose is to power multiple wheels simultaneously: according to the First Law of Thermodynamics, the power from the transmission must be split between the wheels so that energy can be conserved. Assuming symmetry of components entails cutting half of the components away at the split and using the mirror image of the remaining components.

Section 3.1.1.A: Torque Bias and the Symmetry Assumption

Some differentials have the ability to bias torque, sending more torque to one tire than to the other. In truth, this ability is one type of limitation of slip and will only be activated when a tire is slipping. Torque biasing simply prevents the differential from passing enough power to a slipping wheel to cause it to spin up and the other wheel to stop. Since most law-abiding driving in Austin, Texas occurs with all wheels in contact with the ground, and since inclement weather conditions in Austin and Texas rarely cause this driving to include wheels slipping, the model assumes that the torque is split equally between all of the “drive” wheels.

Section 3.1.1.B: The Bearing Load Symmetry Assumption

Bearing efficiency depends not only on the torque it communicates but also the axial and thrust loads it experiences. Since power is being transmitted through gear pairs from one shaft to another, the gear pairs cause the axles to have axial and thrust loads. This, in fact, is why bearings are needed in the powertrain: otherwise, the shafts would rotate about their axes without needing to be constrained by bearings.

The situation of bearings straddling the loading from a gear pair allows for simple calculation of bearing loads based on a fulcrum analysis, or a force and moment balance. This technique is covered in several undergraduate engineering classes from statics all the way to machine design. However, if the bearings are not equally spaced from the gear pair, the bearings would be supporting different loads. Different loads on the bearings would cause the efficiency of the bearings to be different. Hence, part of the assumption of symmetry is to require that bearings are placed equidistant from the gear pair in order to assure that both bearings are loaded axially in the same way.

Furthermore, thrust loads are presumed to be split evenly between the bearings. This assumption is soundly rooted in the theory of deformable solids. As the gear pair loads the shafts with a thrust load, each shaft will compress or elongate, much like a spring. If both shafts going from the spider gears to the bearings are of equal length and cross-section, then the "springiness" of the shafts are the same, causing the thrust load to be split evenly between the bearings.

Section 3.1.2: Operating Temperature Difference

In the absence of information regarding the severity of the driving being undertaken and the ambient temperature around the car, both the ambient temperature and the temperature difference between ambient and operating temperatures must be assumed. These quantities allow the calculation of the operating temperature of the differential and, more importantly, of its lubricant. The viscosity of the lubricant and its associated windage losses are highly temperature-dependent and are governed according to Walther's equation (see Subsection 5.2.2.A). Refer to that section for more information on how viscosity is calculated. For the differential model, the ambient temperature was assumed to be 20 °C, and the temperature difference between ambient and the differential lubricant was assumed to be 50 °C.

Section 3.1.3: Simplified, Laminar Flow for Windage Calculations

When fluid shears, sometimes turbulent eddies are created. The existence of these eddies complicates the calculation of shear stress and resultant windage losses from a lubricant. The Reynolds number is a quantity used to determine whether a shearing fluid will produce turbulence. This determination is based on a correlation to existing data. In fact, several correlations exist for varying enclosure geometries. However, the first attempt at modeling the windage involved the assumption that no turbulent eddies were being created within either the lubricant or the air within the differential housing. A flow free of turbulence is called a laminar flow.

Section 3.1.4: Constant Speed Operation, Neglecting Inertial Effects

When trying to accelerate or decelerate a vehicle, not only do the components load the powertrain through various types of friction, but also the components apply inertial loads upon the powertrain to resist the desired acceleration. The use for which this model is intended calculates the inertial loading effects of the entire powertrain separately from the differential model. This is accomplished by assuming that the differential operates at constant speed over a short period of time. By keeping the time intervals small enough, this numerical approximation can maintain a reasonable degree of accuracy.

Section 3.1.5: Smooth, Dry Pavement With No Loss of Traction

Under conditions when the coefficient of friction between the tire and the road is reduced or under periods of high acceleration demand, the driver of a vehicle can demand a motive force which exceeds the tractive limit of one or more of the drive tires. When this occurs, one of the tires will begin to freewheel. This would cause the differential to execute a complex motion and would activate any limited-slip mechanism inside the differential. Both of these effects result in decreased differential efficiency. However, these events rarely occur in the great state of Texas, as the weather is warm and the vast majority of drivers are kind.

Section 3.1.6: Straight, Level Road With No Turning

If the road were to curve, forcing the driver to turn the steering wheel, the differential would begin to execute a more complex motion than is currently modeled. This motion produces an associated decrease in the differential efficiency. A similar effect could be observed in the center differential of AWD vehicles that are traveling over hilly terrain. The vast majority of roads in Texas are sufficiently straight and flat that neglecting these effects would have only a minor effect on the efficiency calculations.

Section 3.2: Torque/Energy Balance and Applications to Measuring Efficiency

Fuel stores chemical energy; however, converting that chemical energy to translational work is fraught with inefficiencies, as is using the work once converted. After all, the driving cycle dictates the required speed and acceleration, not the fuel consumption, of the vehicle as a function of time. Acceleration is caused by an imbalance of forces, as is known from classical mechanics. These forces must occur as either body or surface forces. The force caused by gravitational acceleration is an example of a body force, as the force is distributed throughout the body. The force caused by aerodynamic drag on the outside of the chassis is an example of a surface force, as the force acts on the surface of the vehicle. Another surface force is the force action/reaction pair between the vehicle tire and the road interface. While other forces may exist, the aforementioned three forces are the only ones typically considered by this type of analysis for simplicity. It is generally assumed that forces perpendicular to the direction of travel sum to zero for a road load examination of straight-line travel. For instance, aerodynamic lift, inertial force, and the road hold the car up against gravity. Hence, only the projections of forces in the direction of travel are generally considered for driving cycles. This is good, since the driving cycle prescribes the acceleration in the direction of travel.

The way the car accelerates against aerodynamic drag is to apply a torque on the tire. That is the job of the powertrain, in fact. The road does not just keep the car from falling through the surface of the Earth, after all. Vehicles take advantage of the phenomenon of friction at the tire-road interface. The engine provides a torque to at least one tire through the rest of the powertrain, which commonly includes a transmission, some bearings, some universal or constant velocity joints, and usually at least one differential. This torque from the engine is the result of liberating energy stored in chemical bonds in the fuel. During combustion, molecules of fuel and air are broken and recombined into exhaust products, which is at a lower chemical potential than the fuel and air mixture. The chemical reaction, sparked by the spark plug and encouraged by turbulence and high pressures in the combustion chamber, is therefore exothermic. The chemical-to-thermal heat release increases the temperature of the working fluid which, in turn, increases the pressure within the cylinder. The elevated pressures act on the piston top surface. This produces a force on the connecting rod which then rotates the crankshaft.

As the driving torque is transmitted through the powertrain, several phenomena cause various components of the powertrain to experience other torques. First and foremost, while the tires grip the road (and even when the tires lose traction), the road applies a force on each of the tires. While the force is balanced by the bearings such that the chassis and powertrain travel at the same speed, the road load resistive torque may only be overcome by the engine and power transmission components. The main focus of the fuel economy model developed for TxDOT Project 0-5974 is to predict the motive force that the engine eventually produces at the tire-road interface, as the drive cycle simulator uses that force to predict the acceleration of the vehicle. However, before the driving (motive) torque is communicated to the drive wheels, other components of the powertrain generally are designed to provide a torque multiplication (although inefficient) while simultaneously decreasing the rotational speed. The sources of torque loss (inefficiency) in the transmission and differential include bearings, seals, gear tooth sliding friction, and windage. An application of Newton's Second Law of Motion for rotation to an input torque (originating from the engine) and a torque applied on an element can yield its output torque.

It is generally obvious how Newton's Second Law of Motion applies to a model of a differential. However, the First Law of Thermodynamics is also applicable for two reasons. Firstly, in the absence of either kinematic relationships or Newton's Third Law of Motion, the First Law of Thermodynamics would resolve the change in either speed or torque, respectively, for a gear pair. Secondly, however, the First Law of Thermodynamics allows an estimation of the efficiency of a differential from temperature data using a heat transfer analysis. If one can assume that all energy dissipated by the torque loss mechanisms in the powertrain become thermal energy, then, under steady operation, the differential's thermal energy loss rate (heat loss) will soon approach the total mechanical energy loss rate across all of the components of the differential. Concessions may be made for other energy inputs, such as solar radiation. Data collection procedures can control or minimize these concerns.

Section 3.2.1: Strain Gauge Technique

The efficiency of a differential can be determined by connecting strain gauges to the three shafts connected to the differential. By knowing the torsional spring coefficient of the shafts, the shear strain of the shafts can be used to determine the torque being transmitted through each of the shafts. By multiplying the torque transmitted through a shaft by the speed at which it is rotating, the power delivered by each shaft can be found. As mentioned before, the efficiency of the differential is simply the sum of the power outputs divided by the power input. In order to generate a useful amount of data, both the speed of differential operation and the torque delivered to the input shaft would need to be controlled. The speed of the differential can be controlled by constraining the output shafts to operate at the speed dictated by the desired input power speed, thus developing a torque throughout the shaft.

Section 3.2.2: Heat Transfer Analysis

Another method for determining the efficiency of a differential is to operate a differential for long enough for it to reach operating temperature. Sufficient knowledge of the heat transfer characteristics of the shafts and the differential housing and the temperature difference across these components would allow for an estimation of the power loss from the differential through heat. Sooner or later, all of the mechanical power that was dissipated becomes heat. Hence, knowing the heat loss at operating temperature and the power input from engine maps and transmission efficiency models would allow a reasonable estimation of efficiency without needing strain gauges.

Section 3.3: Tribology

The study of tribology is dedicated to surfaces rubbing on each other, going back to the Greek root tribos. As surfaces rub on each other, two things occur: friction between and wear of the surfaces. These two factors account for the energy losses associated with rubbing. When an intermediate lubricant is introduced, both friction and wear are affected. Wear is reduced because of the decreasing frequency of contact between the surfaces. Friction is also altered, but it is not necessarily reduced, depending upon the lubrication regime. The subject of tribology is mentioned as a supplement to the gear pair efficiency models.

Chapter 4: Differentials

Automobile manufacturers have used automotive differentials since before World War II. Over the decades since their inception, automotive differentials have seen several improvements and specializations. Henry Jamison Handy produced a wonderful training film for Chevrolet employees in 1937 which explained the fundamental design and operation of a differential to the layperson. That training video has been posted on YouTube and is available to the public. Two screenshots are included here for the reader's convenience, along with a couple of pictures from Dr. Calvert's website and one from Wikipedia.

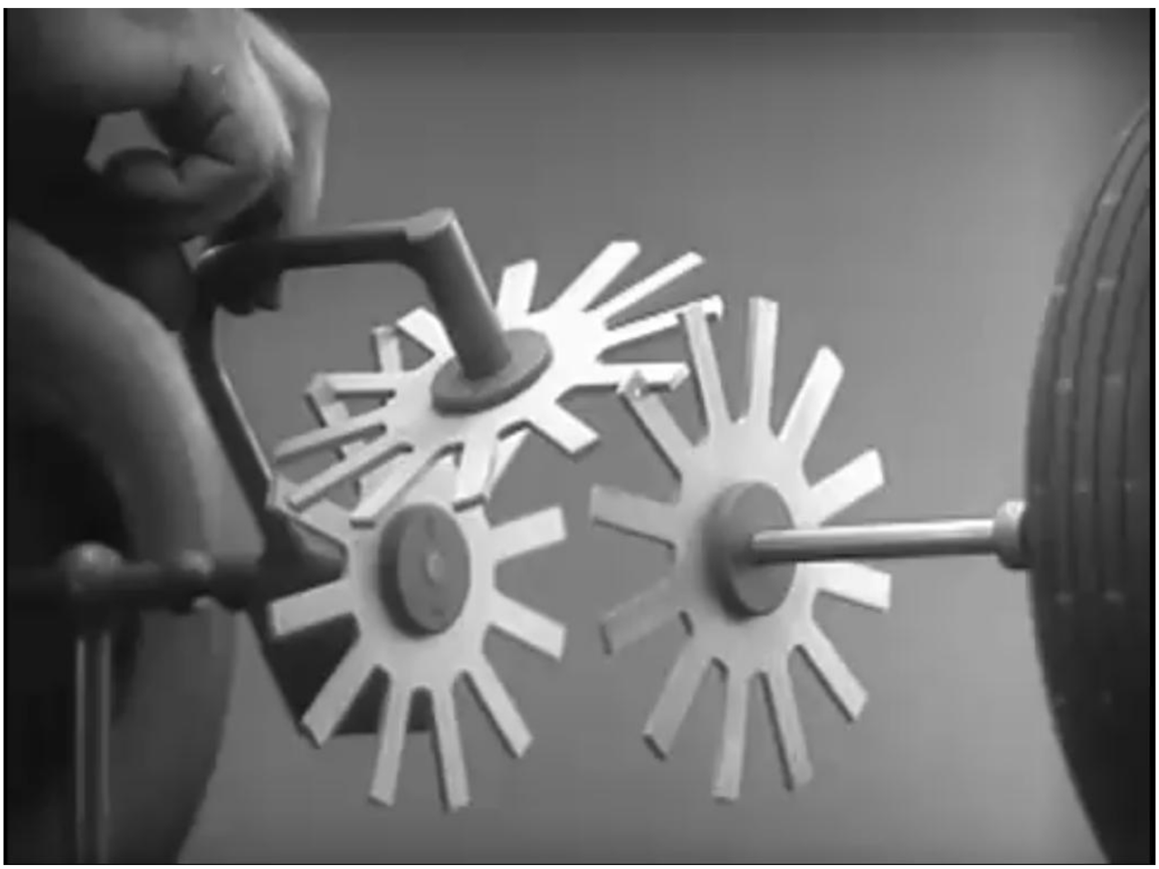


Figure 4.1: One Simplistic Spider Gear Riding on One of the Half-Axles Via the Carrier While Meshing with the Half-Axle Gears, Excerpted from *Around the Corner*.

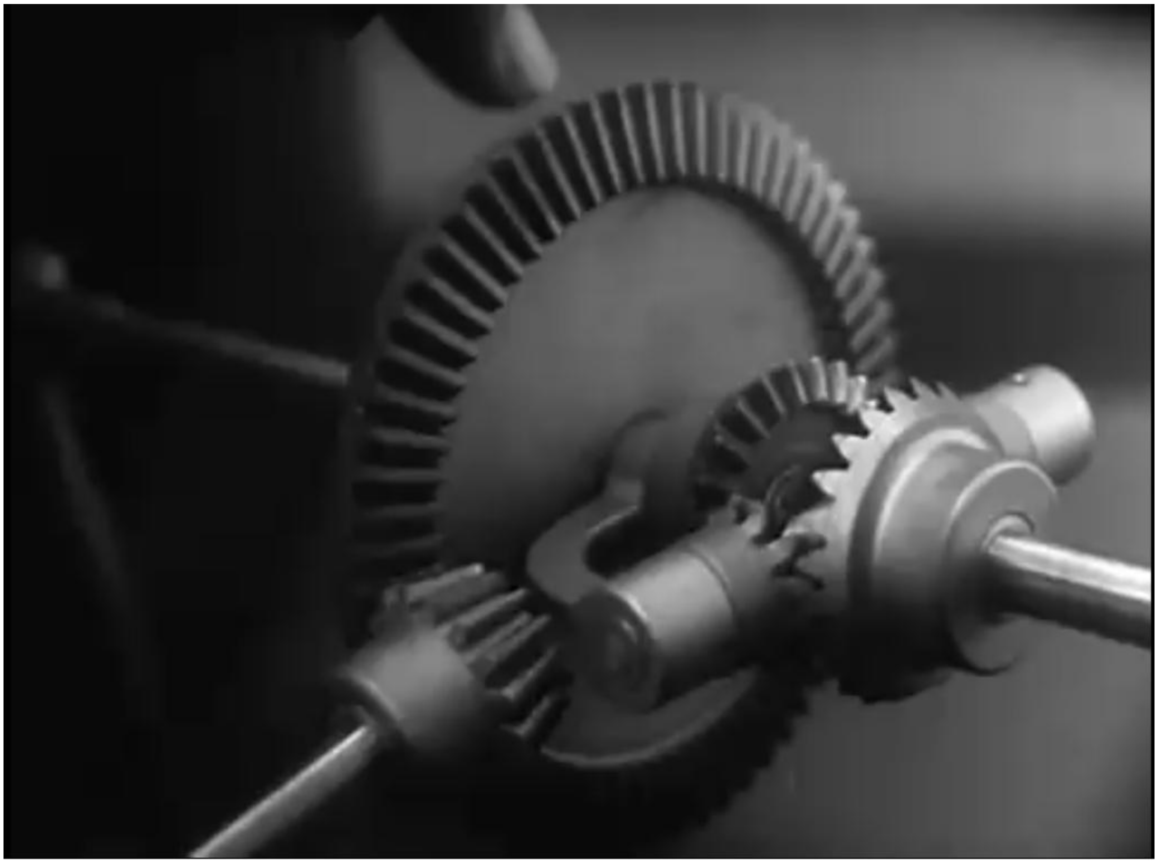


Figure 4.2: Simplest Complete Differential with Bevel Gear Reduction and Two Spider Gears, Excerpted from *Around the Corner*.

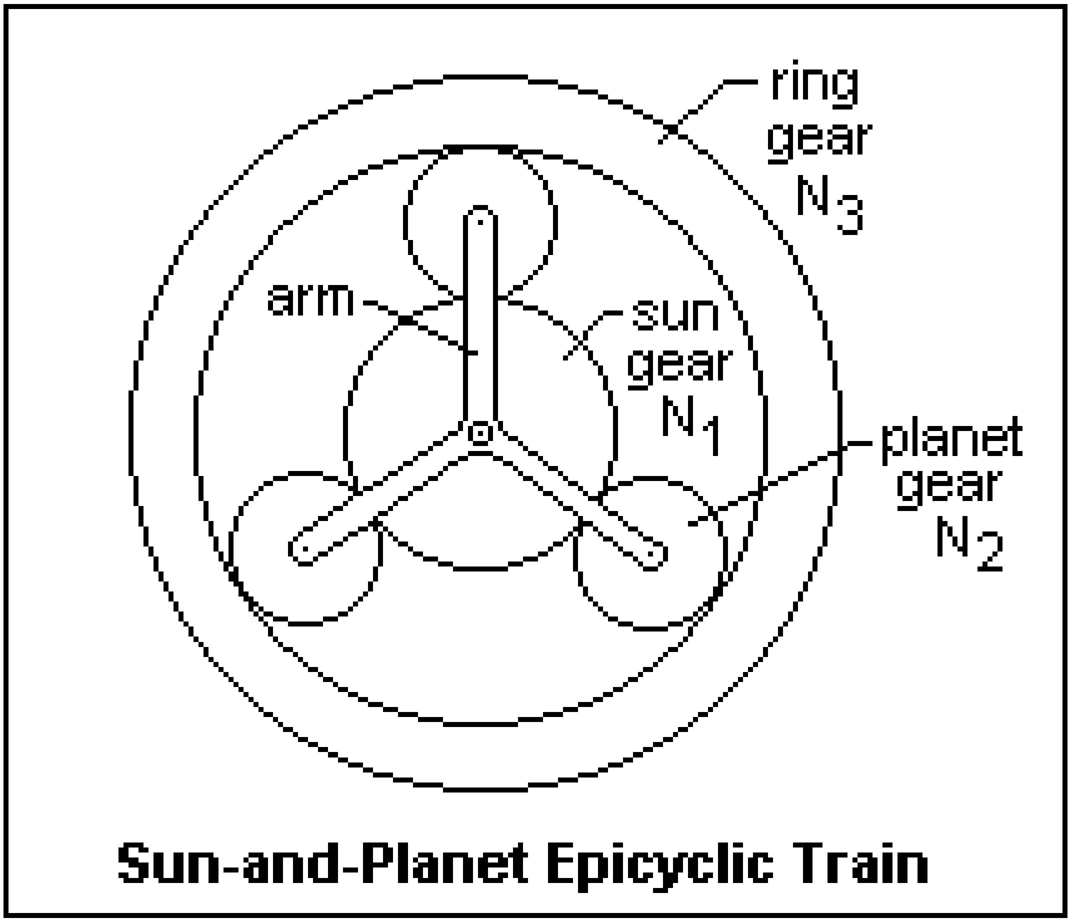


Figure 4.3: A Sun-and-Planet Epicyclic Gear Train Configuration, Excerpted from Dr. Calvert's Website

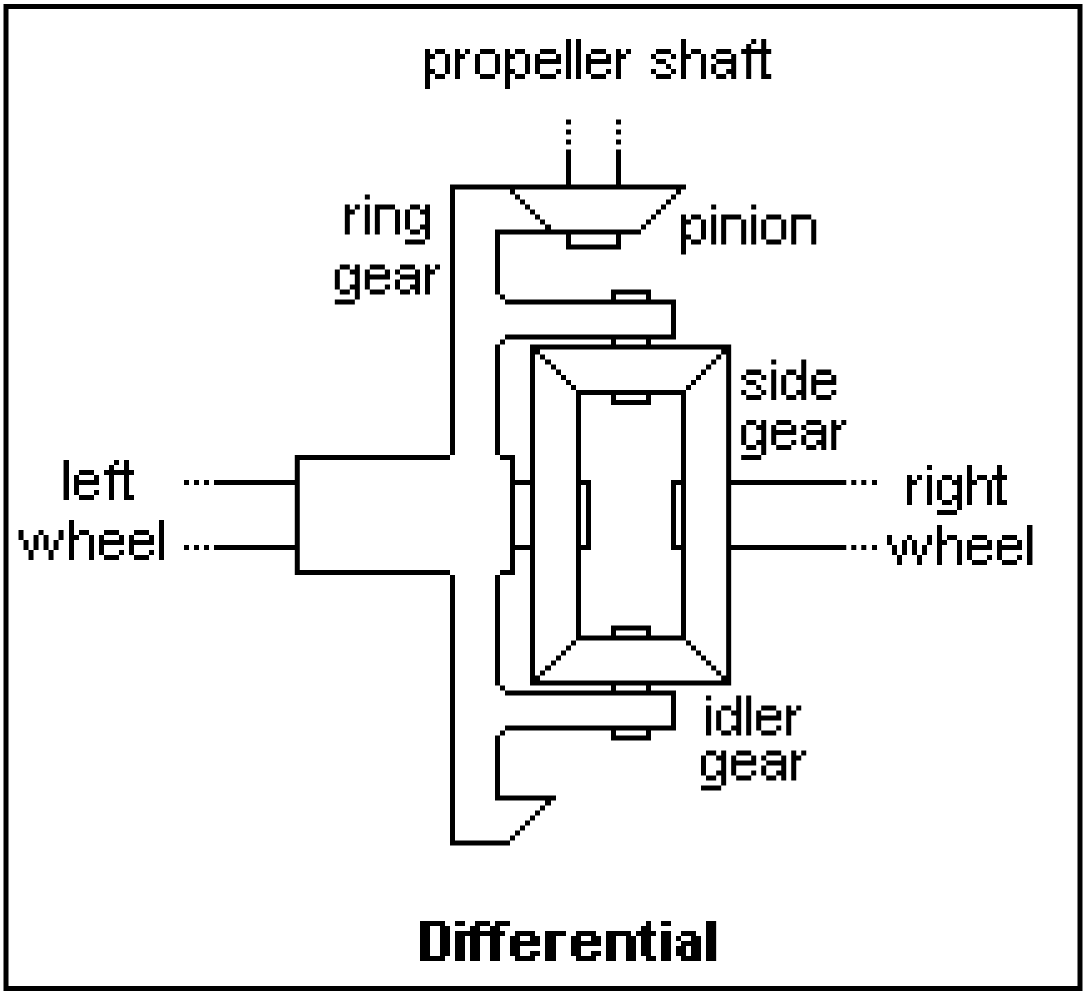


Figure 4.4: A Schematic Representation of an Automotive Differential, Excerpted from Dr. Calvert's Website

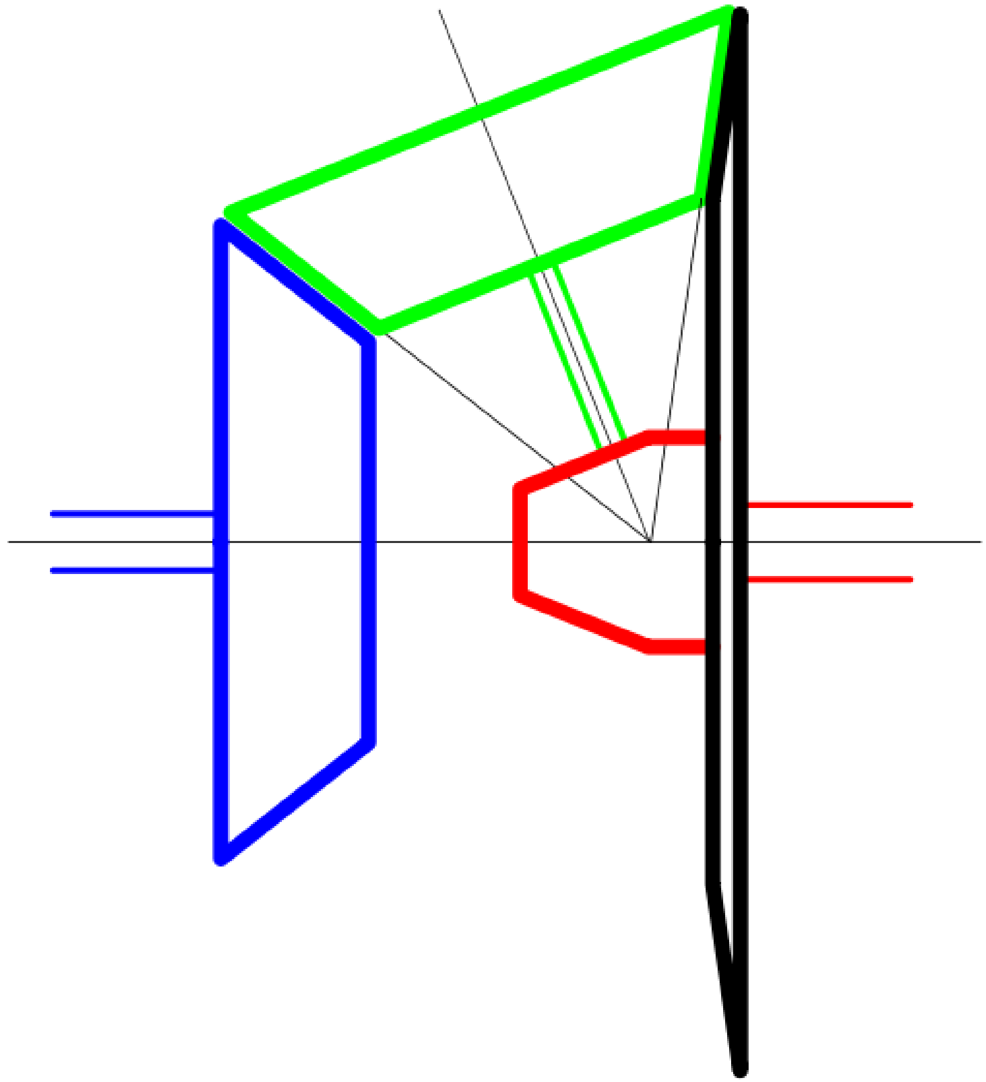


Figure 4.5: An Epicyclic Gear Train Schematic Somewhere Between a Sun-and-Planet Configuration and an Automotive Differential Configuration, Excerpted from the Wikipedia.org Article "Epicyclic Gearing" on 1 March 2012.

The film describes a mechanism which starts from two coaxial output shafts which go to the wheels. Bars are added perpendicular to these two output shafts (as a prelude to gears). A first piece is mounted on one of the shafts by a bearing so that it is free to rotate around the shaft. A second piece is mounted at the end of the first piece such that the second piece has an axis of rotation that always intersects the axis of rotation of the coaxial shafts. A bar is added perpendicular to this second piece (as a prelude to another gear). As limitations are reached, more bars are added with the aforementioned constraints until they are replaced altogether by gears. Differential nomenclature is based on this basic understanding. The first piece is known as the carrier. The second piece and attached bar(s) are known as a spider gear; however, in epicyclic gear nomenclature, these gears would be known as planet gears. The gears on the coaxial shafts would be known in epicyclic gear set nomenclature as dual sun gears.

This configuration is a special case of epicyclic gearing. The classical case of an epicyclic gear set is a sun gear, one or more planet gears, and a star gear. The star gear is an inside-out gear (with teeth on the inside) which holds all of the planet gears against the sun gear. To ensure that the planet gear(s) do not fall out of the gear set, typically a carrier, which is free to rotate coaxially with the sun and star gears, places a shaft through the center of each planet gear. Hence, the planet gear(s) not only rotate(s) but also orbit(s) around the sun gear. The classical case of this has all of the gears in one plane. However, several designs exist for epicyclic gear sets which are not in-plane. The conventional differential design uses bevel gears which bend the planet gear(s) and star gear around until the star gear becomes a second sun gear.

Additionally, multiple types of Torsen (torque-sensing) differential gear sets exist. These Torsen differential gear sets do not require bevel gears. One type uses spur or helical gears that all have parallel axes of rotation. Another type uses worm gears, where the planet gears have axes of rotation that are perpendicular to the axes of rotation of the sun gears. These Torsen differential gear sets have an innate limited-slip capability which solves one of the issues of the conventional differential. Some conventional differentials have been designed to be actively or passively limited-slip, but conventional differentials have no innate source of slip limitation. Other limited-slip designs include electronically sensed and actuated models as well as friction disks.

While differentials nullify the trade-off between acceleration performance and tire cost for ideal road surface conditions, some non-ideal road conditions (such as water, ice, snow, or gravel) may cause one or more of the drive wheels to lose traction. For the conventional differential, a near-equal torque is applied to both drive wheels, so no mechanism exists for slowing down the slipping wheel while powering the other wheels. Limited-slip differentials have, in one way or another, added friction between the output shafts of the differential. As a result, the larger the difference in speed between the wheels connected to a limited-slip differential, the more power is being diverted from the slipping wheel to the wheel that still has traction. Now, limitation of slip counteracts the benefits of a differential. Through one means or another, a limited-slip mechanism will force the differential toward a locked-axle configuration. This forces the speeds of the wheels to be matched and is activated when the conditions favor such behavior. Hence, limited-slip differentials embody a trade-off between the ideal conventional differential and the locked axle.

Digressions aside, the way a differential works can be simple. As the carrier forces the planet gears to orbit the sun gears, in the absence of any outside forces on the sun gears from the wheels, both sun gears (and wheels) will travel at the same speed. If, however, a wheel is being forced to slow down or speed up (such as when turning a corner), that force will be transmitted through the differential in such a way that the wheels are accommodated. The planet gears begin to rotate about their axes as the speed difference develops between the sun gears (and between the wheels). Hence, when the wheels do not lose traction, it is mostly the road that determines what the wheels will do. If the vehicle has lost traction, the road and vehicle speed no longer strictly constrain the behavior of the differential. However, they force the differential's operation toward some steady operation, whether that is that the wheels regain traction or that some other condition occurs, such as one wheel is free-wheeling. In the case of free-wheeling, the power provided upstream of the differential is being fed into the free-wheeling wheel faster than can be dissipated at the tire-road interface, and typically not enough power is being fed to the other wheel. It is this behavior that has necessitated the implementation of limited-slip mechanisms. In fact, before the implementation of limited-slip differentials in passenger vehicles, drivers were trained to demand less power from the engine in those types of situations. Now, technology is surpassing the need for such an artful touch.

With all of these improvements in mind, it can be shown that, under certain assumptions, none of these differences matter when modeling a differential. The major assumptions are straight-line driving and favorable road conditions. The former assumption eliminates turning, and the latter assumption eliminates slipping. Hence, the sun gears will always be turning at the same speed. None of the complicated motions associated with the rotation of the planet gears will occur under these assumptions. Also, the limited-slip mechanisms, whatever they may be, will never become active. Any passive losses from the simple presence of such a mechanism when disengaged are assumed to be negligible in comparison to the other losses. The model introduced in the next chapter will demonstrate that all that remains of a differential is a locked axle connected to the driving shaft by a gear pair. The axles are supported by bearings inside an appropriate housing and sealed from the outside environment where appropriate to retain the lubricant and seal out contaminants. The remaining differences between models are in the powertrain configuration and the weight class of the vehicle. The former determines the orientation and number of components, while the latter determines the dimensions of and relationships between components.

Chapter 5: Present Differential Models with Explanation of Code

A true differential efficiency model would be excellent for the design of automotive differentials. However, most people do not want that degree of sophistication in a differential efficiency model. The modeling elements are completely fundamental; however, the differential models developed for the present thesis are all based on measurements taken from a set of differentials that were determined to be characteristic of the differential type and available for measurement by our research team at The University of Texas at Austin in a cost-effective manner. Techniques developed by several members of the research group have been employed to allow the level of sophistication that the users will want from the model while still making it user friendly.

Section 5.1: Common Modeling Elements

The following sections briefly highlight the concepts used in determining the efficiencies of components that were found in all of the differentials.

Section 5.1.1: Windage

Three types of geometries are of interest for determining the fluid shear drag (windage) losses of a rotating element. These geometries are spherical (since the differential carrier is typically spherical in shape), cylindrical (since shafts and some differential carriers have this geometry), and hypoid gear shapes. Hypoid gears, like bevel gears, look remarkably like frusta of right circular cones or two frusta stuck together, base to base. Hence, the remaining geometry of interest, a hypoid gear shape, is the frustum of a cone. The reader may refer to the following figure for an image of a hypoid gear pair attached to a differential.

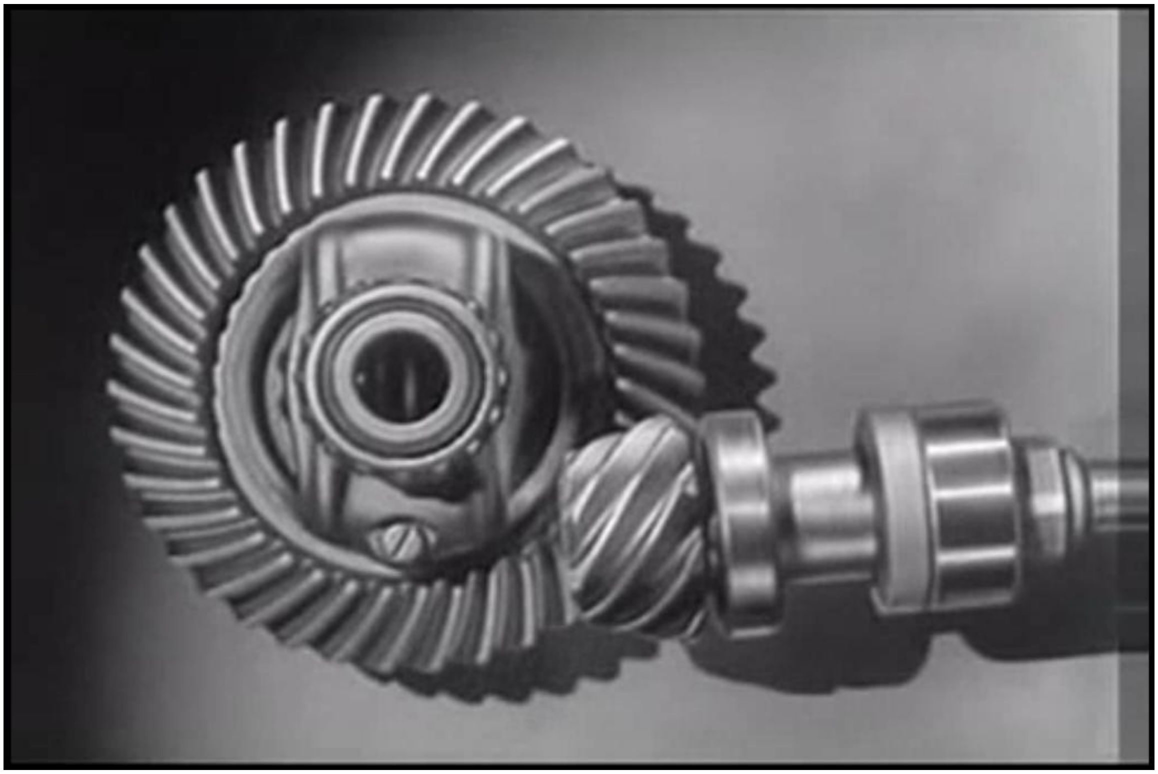


Figure 5.1: A Hypoid Gear Pair Attached to an Automotive Differential, Excerpted from *Around the Corner*.

All of these losses require knowledge of the viscosity of the fluid in which the rotating elements are immersed, as discussed in Subsection 5.2.2.

Section 5.1.1.A: Windage Definition

Some of the energy transmitted by the gear pair is being diverted at the interface between the gears because of friction. Each of the gears is also rotating in a fluid, whether air or differential lubricant, unless the differential housing has been pumped to a perfect vacuum. This motion will cause the fluid to shear (from a continuum perspective; alternatively, one could consider the statistical particle collisions between the gas/liquid particles and the solid gear particles). Fluid shearing is a form of fluid deformation and requires an input of energy in order to do so. In any event, because of fluid shear, gears spinning in fluid also cause some of the energy transmitted to be diverted. Because two different fluids are subjected to shear, the combination of aerodynamic drag (due to the air within the differential housing) and hydrodynamic drag (due to the lubricant "gear oil"), is referred to as windage.

Section 5.1.1.B: Cylindrical Geometry

The losses associated with the rotation of cylindrical geometries within stationary enclosures are readily available in fluid dynamics textbooks. For example, Example 8.2 in Fox et al. (2006) calculates the torque and power dissipation at a given speed for a journal bearing using the assumption of a small gap width "so the flow may be modeled as flow between infinite parallel plates" For the development of the present model, the theory related to flow between infinite parallel plates has been adapted for use in all of the powertrain configurations as appropriate.

Every rotating shaft of constant cross-section is subject to this form of windage, even if it is only in air, because even air has some viscosity. As mentioned previously, the fluid is assumed to shear with laminar behavior. The cylindrical windage torque model is based on the radii of the outside of the shaft and the inside of the enclosure, the lubricant viscosity, the rotating speed, and the length of the shaft within the enclosure.

$$\eta_{w,cyl} = 1 - \frac{\frac{\pi}{4}\mu_{air}\frac{D^3 L}{a}\omega}{\tau}$$

Equation 5.1: Efficiency of Cylinder Considering Windage Losses

Section 5.1.1.C: Spherical Geometry

The losses associated with the rotation of spherical geometries require some calculus to determine. Similar situations are the subject of discussion on websites like Physics Forums: <http://www.physicsforums.com/printthread.php?t=427695>. The surface area of the sphere is exposed to the fluid, so the contribution of shear stress to rotational drag must be integrated over the entire surface area of the sphere; however, to track the moment that the shear stress creates when acting over the surface, the shear stress must be multiplied by the moment arm from the axis of rotation to the infinitesimal surface area element. The resulting integral formulation for fluid shear torque remains a function of the radii of the outside of the sphere and the inside of the shell (differential housing), the lubricant viscosity, and the speed of rotation.

$$\eta_{w,sph} = 1 - \frac{\frac{4\pi^2}{45}\mu_{air}\omega\frac{r^4}{R-r}}{\tau}$$

Equation 5.2: Efficiency of Sphere Considering Windage Losses

Section 5.1.1.D: Frustum Geometry

The losses associated with the rotation of the frustum of a right cone require calculations similar to those required by spherical geometries. The primary difference is in the expression for the curve that is revolved about an axis. Careful consideration of the shear stress and moment arm as integrated across the surface area of the frustum provides an equation for the torque the fluid applies on the rotating frustum. This equation is a function of frustum height, base and peak radii, clearance between frustum and shell, fluid viscosity, and rotational speed. However, an empirically derived equation for the losses associated with the rotation of the frustum of a right cone exist within Dudley's Gear Handbook.

$$\eta_{w,fru} = 1 - \frac{\frac{15}{0.746}\frac{X\mu_{oil}+(1-X)\mu_{air}}{\mu_{air}}\frac{\left(\frac{N}{1000}\right)^2}{\left(\frac{1000}{60}2\pi\right)}\left(\frac{2r_{pitch,pinion}}{100}\right)^4\left(\frac{5L_{fw}}{100}\left(1+\frac{R_f}{\sqrt{\tan(\psi)}}\right)+\frac{2r_{pitch,pinion}}{100}\right)}{\tau}$$

Equation 5.3: Empirical Efficiency of Frustum, Specifically a Hypoid Gear, Considering Windage Losses

Section 5.1.2: Gear Pair

The teeth on gears allow gears to mesh in such a way that, when one gear rotates, another is forced to rotate. When rotating gears are mated in a pair or in more complicated groups, they transmit forces between their teeth. Upon close inspection, one finds that the teeth slide past each other as the gears transmit their forces in motion.

A great thought experiment for the reader requires him or her to place his or her two index fingers so that the insides touch each other completely and the tip of each finger touches the base knuckle of the other. The reader should rest his or her elbows as far from each other as possible on a table, never allowing them to leave the table or move across it. In this way, the reader's index fingers represent teeth on gears whose centers are at the reader's elbows. The reader's forearms represent the moment arms, so to speak, of the gears. The reader should try to raise the lower index finger up while holding both arms rigid, only allowing the arms to pivot about the shoulders and elbows. As the tip of the lower index finger slides across the surface of the upper index finger, the reader can feel the same friction that spur gears must overcome, especially if the reader adds some resistance by lightly opposing the movement with the arm of the upper index finger. The reader may also add some hand lotion to simulate the effects of lubrication, and the reader will notice that the friction felt between index fingers is greatly reduced.

These forces between the gear teeth, balanced by bearings into which the shafts are pressed that carry the gears, impose torques, or moments, on and about the axes of rotation of the gears. The forces on the gears are supported by the axles they are mounted on, which themselves are mounted by bearings to the main structure. However, the slipping behavior of the teeth causes frictional losses, and the rotation of gears in viscous fluid causes windage losses.

Gear pairs serve to exchange a common factor between the torque and speed of a powertrain, though frictional and viscous losses will rob the gear pair of some of its ability to provide the torque to downstream components. This is due to the fact that the ratio of teeth on the gears is the same as the ratio of pitch diameters. While equal and opposite forces develop between the gear teeth, the moment arms are different by the factor that is the gear ratio. Hence, the bigger gear will spin more slowly than the smaller gear but will transmit a greater torque than the smaller gear.

Section 5.1.2.A: Basic Gear Terms

The associated nomenclature for gears is perhaps foreign to most readers. If such is the case, a brief summary of important terms is given below, but more detailed information is available at the Quality Transmission Components website: <http://www.qtcgears.com/Q410/Q420cat.html>.

It bears mentioning that many types of gears exist, based on the placement of the teeth and the orientation of the gears relative to one another as a result of the tooth placement. A substantial (although by no means exhaustive) list of gear types is available in Dudley's Gear Handbook (Townsend, 1991). Of these dozen or so gear types, only two types are of interest in conventional differentials: helical and hypoid gears. Bevel gears also exist in conventional differentials as spider gears, but the assumption of straight-line driving negates their contribution to the efficiency of a differential. The reader is asked to refer to Figures 4.2 and 5.1 for examples of bevel and hypoid gears, respectively. The following figures will illustrate other types of gears.

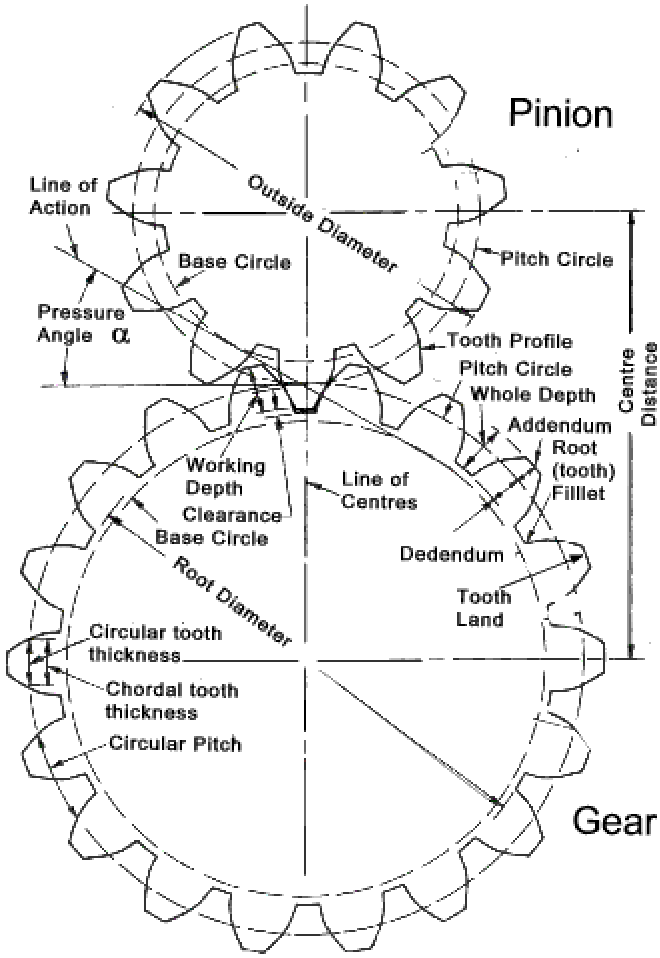


Figure 5.2: Diagram of Spur Gears with Many Helpful Terms, Extracted from <http://www.roymech.co.uk/Useful_Tables/Drive/Gears.html> on 2 March 2012

Figure 5.3: Photograph of Helical Spur Gears, Excerpted from <http://science.howstuffworks.com/transport/engines-equipment/gear3.htm> on 2 March 2012

Figure 5.4: Photograph of Worm Gears, Excerpted from <http://science.howstuffworks.com/transport/engines-equipment/gear5.htm> on 2 March 2012

The simplification of gears for analysis generally requires knowledge of a pitch radius/diameter which is somewhere between the radius at the tips of the teeth and the radius at the roots of the teeth. The pitch radius/diameter is the radius/diameter of the solid shape that would replace gear teeth with cylindrical surfaces if one were to rely on friction rather than strict kinematics to transmit a torque and rotating speed.

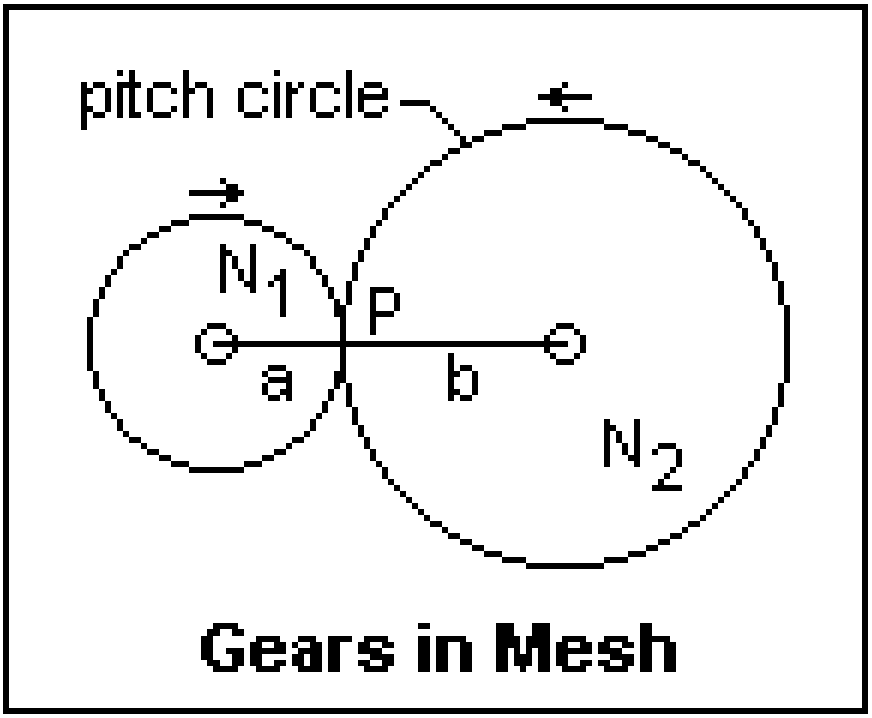


Figure 5.5: Diagram Depicting the Pitch Radii of Meshed Gears, Excerpted from Dr. Calvert's Website

For instance, two spur gears could be replaced by "sticky" cylinders whose radii are the pitch radii of the spur gears. However, unlike such a configuration with cylinders, the spur gears will not slip with respect to one another unless a tooth breaks off. Gear pair efficiency models use the pitch radius instead of the outer or inner radius because the action of rubbing, sliding, and force transmission happens at or near the pitch radius regardless of the size of the teeth. The reader is asked to refer to the empirical model for the efficiency of a hypoid gear rotating in viscous fluid, Equation 5.3, for an instance of the usage of pitch radius. A crude approximation of pitch radius can be determined by taking the mean of the inner and outer radii. However, the true pitch radius is dependent upon the profile of the gear tooth.

All gears have a normal pressure angle ϕ_n because the teeth do not jut straight out of the gear. This causes the gears to naturally repel each other through the use of inclined planes. Furthermore, forces between the teeth give rise to bending moments on the shafts the gears are pressed or keyed onto. Some gears have teeth that are angled in such a way that they develop a helix angle ψ or γ, depending upon notation. This will cause the gears to pull or push on the shafts on which they are mounted by developing an axial load on the shaft, in contrast to the normal pressure angle, which causes shafts to bend. Both of these angles affect the load transmitted at the tooth-to-tooth interface, affecting the energy required to slide the teeth with respect to one another.

Two gears may mesh despite having a different number of teeth. The ratio between the number of teeth is known as the gear ratio G (and is equal to the ratio of pitch radii). Due to kinematic relationships, the gear ratio from one gear to another is the inverse of the speed ratio in the same direction. Furthermore, neglecting inefficiencies, the gear ratio from one gear to another is equal to the torque ratio in the same direction. Typically, the driving gear has fewer teeth, carries less torque, and spins faster than the driven gear.

$$G = \frac{n_{pinion}}{n_{gear}} = \frac{N_{gear}}{N_{pinion}}$$

Equation 5.4: Definition of Gear Ratio in Terms of Tooth Number and Gear Speed

All of these terms are quite common in gear design and are quite useful in predicting the efficiency of gear pairs. Another term is the face width L_{fw} of the tooth, which is related to the thickness of the gear and the helix angle. These terms will be discussed further in the following subsections.

Section 5.1.2.B: Differential Gear/Final Drive/Axle Ratio

The differential gear ratio, also known as the final drive ratio or axle ratio, is the gear pair that transmits torque from the transmission to the spider gear carrier inside the differential. It is the only gear ratio of interest in the differential models.

Section 5.1.2.C: Helical Gears (FWD and AWD only)

Helical gears are an improvement on spur gears because of the decrease in noise. A helical gear pair exists in the differential of a transaxle of a front-wheel-drive (FWD) passenger car. Some all wheel drive (AWD) vehicles have a transaxle as well.

Many models exist for the efficiency of a helical gear pair. Dudley's Gear Handbook (Townsend, 1991) is an excellent place to start. Subsection 12.3.3 of that reference, entitled "Single-Helical Gears," provides the reader with an "equation for the percent power loss." A proper understanding of the nomenclature can bridge the gap between the published equations provided or measurements collected and a real-world understanding of the significance of those quantities. In addition to Townsend's book, Appendix C in Matthews' (2011) text includes the Buckingham equation for gear pair efficiency. Another good reference is a paper by Diab and coworkers (2004).

$$\eta_{gp,hel,w,pin} = 1 - \frac{1.069 \times 10^{-4} C_E \dfrac{1 + 2.3 L_{fw}}{R_{p,pin}} \omega^{1.85} OD_{pin}^{4.7} \left[X \left(\rho_{oil} \nu_{oil}^{0.15} \right) + (1 - X)\left(\rho_{air} \nu_{air}^{0.15} \right) \right]}{\tau}$$

Equation 5.5: Efficiency Calculation of the Effect of Windage on the Helical Pinion

$$\eta_{gp,hel,f} = 1 - \frac{\mu_{f,gp}}{2 \dfrac{C}{1+G} \cos(\alpha)} \left| 1 - \frac{1}{G} \right| \frac{C \sin(\alpha)}{1+G} \frac{1+G^2}{|1+G|} \left(1 - \varepsilon + \varepsilon^2 \right)$$

Equation 5.6: Efficiency Calculation of the Effect of Gear Friction on the Helical Gear Pair

$$\eta_{gp,hel,w,gr} = 1 - \frac{1.069 \times 10^{-4} C_E \frac{1 + 2.3 L_{fw}}{R_{p,gr}} \omega^{1.85} \left[OD_{gr}^{4.7} - \left(2R_{i,gr} \right)^{4.7} \right] \left[X \left(\rho_{oil} \nu_{oil}^{0.15} \right) + (1 - X) \left(\rho_{air} \nu_{air}^{0.15} \right) \right]}{\tau}$$

Equation 5.7: Efficiency Calculation of the Effect of Windage on the Helical Driven Gear

Section 5.1.2.D: Hypoid Gears (all except FWD)

Helical gears are one simple improvement descended from spur gears, the easiest gears to understand. Hypoid gears, however, are something like a cross between a helical, bevel, and worm gear pair. The teeth of a hypoid gear are beveled; that is, they are not perpendicular to any surface of a typical cylindrical disk (unlike spur, helical, and worm gears). However, the rotational axes of the hypoid gears are neither parallel nor intersecting, they are skewed. The teeth themselves of a hypoid gear are also curved like the teeth of helical gears. Handy's 1937 film ("Around the Corner", http://www.youtube.com/watch?v=yYAw79386WI, accessed 11/3/2011) also covers the improvement of hypoid gears over straight bevel gears. The film cites the reason for the use of hypoid gears, which are not as efficient as bevel gears, because of the increased leg room available to the passengers because the transverse axle could be lowered by a few inches using hypoid gears. Since hypoid gears are typically more efficient than worm gears, worm gears appeared less desirable. Again, a trade-off resulted in an engineering decision.

Fewer models exist for the efficiency of a hypoid gear pair when contrasted with helical gear pairs. Again, Dudley's Gear Handbook (Townsend, 1991) is an excellent place to start. Another good reference about hypoid gear pairs is by Hai (2005).

$$\eta_{gp,hyp,w,pin} = 1 - \frac{\frac{15}{0.746} \frac{X\mu_{oil} + (1-X)\mu_{air}}{\mu_{air}} \frac{\left(\frac{N}{1000}\right)^2}{\left(\frac{1000}{60} 2\pi\right)} \left(\frac{2r_{p,pin}}{100}\right)^4 \left(\frac{5L_{fw}}{100}\left(1 + \frac{R_f}{\sqrt{\tan(\psi_{pin})}}\right) + \frac{2r_{p,pin}}{100}\right)}{\tau}$$

Equation 5.8: Efficiency Calculation of the Effect of Windage on the Hypoid Pinion

$$\eta_{gp,hyp,f} = \frac{\cos(\phi_n) + f \tan(\psi_{gr})}{\cos(\phi_n) + f \tan(\psi_p)}$$

Equation 5.9: Efficiency Calculation of the Effect of Gear Friction on the Hypoid Gear Pair

$$\eta_{gp,hyp,w,gr} = 1 - \frac{\frac{15}{0.746} \frac{X\mu_{oil} + (1-X)\mu_{air}}{\mu_{air}} \frac{\left(\frac{N}{1000}\right)^2}{\left(\frac{1000}{60} 2\pi\right)} \left(\frac{2r_{p,gr}}{100}\right)^4 \left(\frac{5L_{fw}}{100}\left(1 + \frac{R_f}{\sqrt{\tan(\psi_{gr})}}\right) + \frac{2r_{p,gr}}{100}\right)}{\tau}$$

Equation 5.10: Efficiency Calculation of the Effect of Windage on the Hypoid Driven Gear

Section 5.1.3: Bearings

Bearings are required to constrain the motion of a shaft to pure rotation while still allowing it to rotate relative to its mount. In the case of a differential, bearings force the shafts to align in such a way that the gears that should be in contact are in contact. Bearing losses occur because of friction and wear. Bearing loss calculations are also discussed very well in Khonsari and Booser (2008).

$$\eta_b = 1 - \frac{M_P + M_L + M_S}{\tau}$$

Equation 5.11: Decomposition of the Efficiency of a Bearing into its Constituent Torques

The following figure illustrates a typical cylindrical roller bearing.

Figure 5.6: Photograph of Tapered Roller Bearings Without Inner Races, Excerpted from the Timken Website on 2 March 2012

Section 5.1.3.A: Types

Many types of bearings exist. Two main categories of bearings are roller element bearings and fluid film bearings. Roller element bearings are of interest for this model; and fluid film bearings are not present in automotive differentials. Roller element bearings are further categorized by the type and arrangement of rolling element. A bearing may have one or more rows of rolling elements. Furthermore, the rolling elements may take many shapes, such as spheres, cylinders, slender cylinders (or pins), cones, or barrels.

Section 5.1.3.B: Types Used in the Present Differential Models

The present differential models used single-row tapered roller bearings. Tapered rollers are like truncated cones, or frusta. The moment applied by any bearing is the sum of three moments from different sources of loss. One of these moments is load-dependent, while another is speed-dependent. The moments are also functions of diameters, coefficients of friction, and lubricant viscosity. The reader is asked to refer to Equation 5.11 to view the relationship between moments imparted on the bearing and the corresponding bearing efficiency. The reader is also referred to Khonsari and Booser's text for information on the individual moment calculations. The following information from Khonsari and Booser's tables 15.1 through 15.3 were used in the calculations: oil bath lubrication was presumed, 0.0018 was used for μ, 8 was used for the lubrication friction factor f_L, 20 was used for factor f_1, and 25 was used for factor f_2.

Section 5.1.4: Seals

Seals are required to keep lubricant inside with the gears and bearings while keeping contaminants out. In fact, some bearings have built-in seals; for others, this is not necessary (and is cheaper not to have). The model for seal efficiency is based on that in Appendix C of Matthews' (2011) text. The seal model is based solely on geometry and an adjustable factor F'_{tan}, which was presumed to be 135 N/m in most cases.

$$\eta_s = 1 - \frac{2\pi r^2 F'_{\tan}}{\tau}$$

Equation 5.12: Efficiency Calculation for Seal Losses

The following figure illustrates a typical automotive seal.

Figure 5.7: Photograph of Automotive Seals, Excerpted from the Timken Website on 2 March 2012

Section 5.2: Common Parameters

The following sections discuss parameters that do not belong to any particular component or need further discussion.

Section 5.2.1: Peak Torque

The peak torque available to the differential is a product of several factors. First, the engine produces a peak torque at its optimal range. Under the most severe conditions, the engine can supply this torque to the transmission when the transmission is in its lowest gear, or highest torque multiplication. If the transmission is an automatic, the torque converter will first multiply the torque before it reaches the transmission gearbox. Knowledge of all of these factors allows generation of a peak torque value which can be used in a method proposed by a member of this research team, PhD candidate Kyung Jin Kim, to compare the efficiency of one differential with the efficiency of another differential.

Section 5.2.1.A: Kyung Jin Kim's Method

In a few words, PhD candidate Kyung Jin Kim's method scales the differential input torque by the highest possible input torque that it could see for that particular vehicle. Scaling the input torque generates a torque fraction that is halfway to converting a torque value for one vehicle's differential to the equivalent torque value for another vehicle's differential. Once the torque fraction is provided to the differential model, the torque fraction is converted back into a torque based on the torque capacity of the differential upon which the model is based (and the measurements were made).

Section 5.2.1.B: Peak Engine Torque

Data will exist for the wide-open throttle (WOT) torque that an engine can produce at any speed if it was calibrated on a dynamometer. This WOT torque curve is generally not available to the user of the fuel economy model produced for TxDOT Project 0-5974. The maximum torque produced over the entire range of engine speeds at wide-open throttle is the engine's peak torque. Although this torque is often available to the user of this fuel economy model, it is not always available. Thus, the fuel economy model user is required to input the maximum brake power and the corresponding engine speed. From these, the fuel economy model calculates the corresponding WOT torque. Although this is not the peak torque, it is generally not much smaller. As the maximum torque available from the engine increases, the transmission and differential will need to be designed more robustly in order to offer the same gear ratios.

Section 5.2.1.C: Maximum Torque Converter Torque Ratio

The torque converter is the component of an automatic transmission that allows the engine to idle while the wheels are stopped and the transmission is still in gear. Vehicles with manual transmissions must shift the transmission into neutral or keep the clutch pedal depressed before coming to a stop, or the engine will stall. Torque converters are capable of immense changes in speed with more limited effects on torque. The torque ratio, the torque multiplication by an automatic transmission's torque converter, still needs to be taken into account. This is information that the user of the Fuel Economy Model will not be able to supply. When data were not available for peak transmission torque ratio, a peak torque ratio of three was used.

Section 5.2.1.D: Maximum Transmission Gear Ratio

As mentioned previously, the transmission multiplies the torque received from the engine. For an automatic transmission, this torque multiplication occurs in both the torque converter and the gearbox. The maximum transmission gear ratio provides the maximum torque multiplication factor by the gears of the transmission's gearbox.

Section 5.2.2: Lubricant Type

Oil viscosity ranges are specified by their grade. Oil viscosity ranges are specified by two numbers: the first is a cold viscosity range, and the second is a hot viscosity range. Kyung Jin Kim, PhD candidate, introduced the differential model's author to Walther's equation, which is used to calculate the viscosity of oil as a function of temperature from two points. Walther's equation is discussed below.

Section 5.2.2.A: Walther's Equation

Walther's equation relates the kinematic viscosity of a fluid to its temperature. Three constants exist within the equation that must be fit for each type of lubricant. The equation itself is (Walther, 1933).

$$\log_{10}\left(\log_{10}\left(\nu+C\right)\right)=A+B*\log_{10}\left(\mathrm{T}\right)$$

Equation 5.13: Walther's Equation

Great care must be used in determining the value of constant *C* to ensure that the sum is greater than 1. Otherwise, the equation is no longer useful. For all of the liquid lubricants considered, a value of 0.8 centiStokes was used for C. Equation 5.13 provides the oil's kinematic viscosity, ν, in centistokes given the temperature, T, in degrees Fahrenheit. The solution used for kinematic viscosity follows in Equation 5.14.

$$\nu(T)=10^{10^{A+B\log_{10}(T)}}-0.8;\begin{bmatrix}A\\B\end{bmatrix}=\begin{bmatrix}1 & \log_{10}(T_1)\\1 & \log_{10}(T_2)\end{bmatrix}^{-1}\begin{bmatrix}\log_{10}(\log_{10}(\nu(T_1)+0.8))\\\log_{10}(\log_{10}(\nu(T_2)+0.8))\end{bmatrix}$$

Equation 5.14: Solution to Walther's Equation for Kinematic Viscosity

Section 5.2.2.B: Air

Air is the only fluid of interest from the perspective of the differential model for which Walther's equation does not apply. Instead, the 1972 empirical Sutherland model, as reproduced in and using constants found in Fox, McDonald, and Pritchard's 2004 text, was used and is.

$$\mu=\frac{b\mathrm{T}^{1/2}}{1+S/\mathrm{T}}$$

Equation 5.15a: Sutherland Model for Dynamic Viscosity of Gases

The values used for b and S are as follows (Fox et al., 2004).

$$b = 1.458 \times 10^{-6} \frac{\text{kg}}{\text{m-s-K}^{1/2}}$$

Equation 5.16: The b Coefficient for Air Corresponding to the Sutherland Model for the Dynamic Viscosity of Gases

$$S = 110.4 \text{ K}$$

Equation 5.17: The s Coefficient for Air Corresponding to the Sutherland Model for the Dynamic Viscosity of Gases

Therefore, when applied to air Equation 5.15a simplifies to:

$$\mu_{\text{air}} = \frac{1.458 \cdot 10^{-6} \text{T}^{1/2}}{1 + 110.4/\text{T}} \frac{\text{kg}}{\text{m-s}}$$

Equation 5.15b: Sutherland Model for Dynamic Viscosity of Air

Section 5.2.2.C: ATF

Automatic transmission fluid (ATF) is used in transaxles, so the FWD's differential is exposed to ATF. If the center differential of an AWD vehicle is located in a transaxle assembly, then the AWD's center differential is also exposed to ATF. ATF tends to be less viscous than the other gear oils examined. Walther's equation is used to determine the viscosities of ATF. As previously mentioned, a value of 0.8 cSt has been presumed for C. A and B are taken to be approximately 7.9 and -3.1 K^{-1}, respectively. Furthermore, to ascertain ATF's dynamic viscosity at temperature, an approximation was used to determine the density of the ATF considering thermal expansion.

$$\rho(T) \approx \rho(T_0)[1 + \alpha_V (T - T_0)]; \alpha_V \approx 7 \times 10^{-4} {}^\circ C^{-1}; \rho_{ATF,70^\circ C} \approx 822 kg/m^3$$

Equations 5.18: Density Model and Result for ATF

Section 5.2.2.D: 75W90 and 80W90

Both of these typical differential gear oils are substantially more viscous than engine oil, as denoted by the higher numbers. The nomenclature for these oils is defined by the Society of Automotive Engineers. Data were reproduced in Fox, McDonald, and Pritchard's 2004 text from SAE standards J300, J306, and J311, all published in 1987. As previously mentioned, a value of 0.8 cSt has been presumed for C. For 75W90, A and B are taken to be approximately 7.9 and -3.0 K^{-1}, respectively. For 80W90, A and B are taken to be approximately 8.6 and -3.3 K^{-1}, respectively. Again, the densities of the gear oils were needed and were found to be as follows.

$$\rho_{75W90,70°C} \approx 827 kg/m^3 ; \rho_{80W90,70°C} \approx 854 kg/m^3$$

Equations 5.19: Density Model Results for Gear Oils

Section 5.2.3: Immersion Fractions

The discussion of the fundamental engineering discipline of tribology revealed that frictional losses and wear can be affected by adding a lubricant as a third body in the friction regime. Of course, since the nature of gear oils is to behave like a liquid by conforming to the geometry of its container and being constrained by geometry, a method must be devised to continuously provide the gears with their lubricant. Sealed bearings are more forgiving, being packed with a grease instead of an oil. In a similar way to how bearings scavenge grease, gear pairs can scavenge oil that has run down to the bottom of the gear enclosure as long as at least one of the gears of the pair spins through the puddle of oil in such a way that all of the teeth become entirely coated with oil before reaching the teeth of the other gear. However, this requires that the (typically) larger gear experiences fluid viscous losses across a relatively large moment arm. Hence, the less of the gear that is immersed while still lubricating the gear pair, the better. The optimal configuration for this is called "dipped" in the models for differentials. The value associated with this immersion fraction may vary with design, but was typically observed to be somewhere on the order of 20%. The reader may verify this the same way the author did by locating an older vehicle, laying on the pavement, and observing the placement of the differential lubricant fill plug on the differential's outer shell.

Oil pumps are used in transmissions to spray oil onto all of the gears to remove this viscous loss, though oil pumps have a fair number of moving parts that have frictional and viscous losses associated with their operation. However, the engineering tradeoff favors oil pumps for transmissions, as the majority of the automotive market demonstrates. The reader will recall that FWD and some AWD vehicles have a differential located in the transaxle assembly which also contains a transmission. As the transaxle also generally includes an oil pump, the differential in a transaxle can take full advantage of the oil pump and avoid the viscous losses with relatively low energy cost in utilizing a slightly stronger oil pump when compared with the energy losses associated with viscous dissipation of partially immersing one of the gears. The models for the differentials call this configuration "sprayed from oil pump." Even though the gears are technically not immersed, an equivalent immersion fraction (the fraction that is actually experienced of total windage the gear would experience if fully immersed in the lubricant within its enclosure) must be used to accommodate losses in the oil pump. For manual transmissions, this number is taken to be 3%, as mentioned in Dr. Matthews' text's Appendix C.4: Windage Losses for Gears. This number has been extended to include the differential of a transaxle as well.

Two of the immersion fractions introduced were called dipped and sprayed from oil pump. Two other immersion fractions exist for testing purposes. Unlike with dipped and sprayed from oil pump fractions, the values of the other two fractions are easily tractable. The other two immersion fractions are "halfway" and "totally" immersed, corresponding to 50% and 100% immersion, respectively.

Section 5.3: Light-Duty FWD

The University of Texas at Austin allowed the disassembly and measurement of a spare transaxle from their 2005 Challenge X car, a Chevrolet Equinox. The transaxle is pictured in Appendix C. Since research team member Kyung Jin Kim's transmission model ends at the transmission output shaft, the first component considered in the light duty transaxle differential model is the helical gear pair. As previously mentioned in Subsection 5.1.2, the efficiency of a gear pair may be divided into three parts: the gear pair tooth friction follows the windage on the pinion gear, while the windage on the driven gear follows the gear pair tooth friction. After passing through the gear pair, the power transmission path flows through the spider gear carrier, which is assumed to have spherical shell geometry and is exposed to windage and bearings. The power transmission path splits from the carrier onward as discussed in Subsection 5.1.1.C. Splined shafts extend from the carrier. After the shaft windage, the power transmission path flows past a seal and extends through a region of the shaft which may be recessed within a windage shield. Those shafts extend to the wheels through a couple of universal or constant velocity joints and a bearing for each wheel, but their analysis is not included in the analysis of the differential. In the overall Fuel Economy Model, the losses in the wheel bearings and CV joints or U-joints are assumed to be negligible. Measurements from all of the aforementioned components that are within the bounds of consideration will follow, as will a description of the determination of the scaling parameters used for the Equinox's transaxle differential. First, however, it is important to note that, while the dimensions for other transaxle differentials may differ from those measured from the Equinox, the overall architecture is assumed not to differ substantially amongst the transaxle differentials. The scaling parameters that will follow are used to accommodate the differences in transaxle differential dimensions using the method proposed by co-investigator Kyung Jin Kim.

The following measurements were taken from the differential of the transaxle of the 2005 Chevy Equinox. For the helical gear pair, the numbers of teeth of the gears were taken and the gear ratio of 61:23 derived from them. The outer circumference of the driven helical gear was measured to be 26-7/8 inches. The tooth height was measured to be 5/16 of an inch. From those two pieces of information, the pitch radius of the driven gear, $R_{p,d}$, was calculated given the outside diameter of the driven gear, $d_{o,d}$, and the tooth height of the driven gear, h_{tooth}, via the following equation.

$$R_{p,d} = \frac{1}{2}\left(d_{o,d} - \frac{h_{tooth}}{2}\right)$$

Equation 5.20: Approximation of the Pitch Radius of the Driven Gear

The pitch radius of the driving, or pinion, gear ($R_{p,D}$) requires only knowledge of the pitch radius of the driven gear and the gear ratio, G, via the following definition of the gear ratio.

$$G = \frac{R_{p,d}}{R_{p,D}}$$

Equation 5.21: Definition of Gear Ratio in Terms of Pitch Radii

The gear tooth face width, *FW,* and gear thickness, *GW,* were measured to be 1-3/8 inches and 1-7/32 inches, respectively. This allowed for an estimation of the helix angle, γ, of the gears using trigonometric relationships.

$$\cos(\gamma) = \frac{GW}{FW}$$

Equation 5.22: Definition of Helix Angle γ (or ψ, Depending Upon Notation)

The bearings were observed to be tapered roller bearings that had a circumference of approximately 10 inches. The diameter of the tapered rollers was on the order of 1/4 of an inch. The shafts pressed into the bearings had a circumference of 3-5/8 inches and existed inside an enclosure of diameter 2-3/8 inches between the bearings and the seals. The seals were measured as being 8 inches apart, while the diameter of the spherical spider gear carrier was measured to be 5-1/2 inches, which indicates a similar bearing spacing. The spherical spider gear carrier exists inside a spherical shell whose inner diameter was measured to be 6 inches. These measurements are sufficient to define or reasonably approximate the missing parameters in the component models that go into the light duty FWD differential model.

The modeling effort involves placing all of the components in series, assuming symmetry when necessary. The efficiencies of each of the components are calculated based upon values obtained from upstream components. The efficiency of the differential of the light-duty front wheel drive transaxle, η_{diff,LD_FWD}, is the product of the component efficiencies.

$$\eta_{diff,LD\_FWD} = \eta_{gp,hel} * \eta_{wind,sph} * \eta_{bear} * \eta_{wind,cyl,1} * \eta_{seal} * \eta_{wind,cyl,2}$$

Equation 5.23: Decomposition of Light-Duty Front Wheel Drive Differential's Efficiency into Constituent Component Efficiencies

The right-hand side of Equation 5.23 must be evaluated one component at a time from left to right. Each efficiency will modify the torque as the torque travels through the powertrain before the torque gets to the next component. As such, the model works by modifying and passing the torque from component to component.

Section 5.4: Light-Duty RWD

Inland Truck Parts of Austin graciously allowed the measurement of a generic 1500-series, or 3/4 ton pickup truck, rear axle differential. The knowledge of the load capacity of a typical truck in that classification allowed approximation of the scaling factors that are necessary for generalization of the model. The following equation presents the efficiency calculation with the order of the components considered.

$$\eta_{diff,LD\_RWD} = \eta_{seal,1} * \eta_{bear,1} * \eta_{wind,cyl,1} * \eta_{gp,hyp} * \eta_{wind,sph} * \eta_{wind,cyl,2} * \eta_{bear,2} * \eta_{seal,2}$$

Equation 5.24: Decomposition of Light-Duty Rear Wheel Drive Differential's Efficiency into Constituent Component Efficiencies

Section 5.5: Light-Duty 4WD with Transfer Case

One of Jeep's powertrain configurations, called the YJ, is very popular among Jeep owners, according to a local Jiffy Lube employee. This powertrain configuration includes a transfer case. Jeep publishes specifications for all of its parts, as Jeep owners frequently break and repair their own vehicles. As such, it should be fairly easy to obtain the measurements needed to complete a future model.

A first approximation of a transfer case is a two-speed chain drive system (to provide 4HI and 4LO). While the old method for operating 4WD systems was to connect/disconnect the front wheels to/from the hubs, the new method is to engage/disengage the chain drive system that powers the front wheels. The chain drive system is presumed to have 97-99% efficiency, as evidenced in Shigley's *Mechanical Engineering Design*. The following equation presents the efficiency calculation with the order of the components considered and with respect to models for the other components of the differential.

$$\eta_{diff,LD\_4WD} = \eta_{gb} * \eta_{tc} * \eta_{diff,LD\_RWD}$$

Equation 5.25: Decomposition of Light-Duty Four Wheel Drive Differential's Efficiency into Constituent Component Efficiencies

The product of η_{gb}, the gearbox efficiency, and η_{tc}, the transfer case efficiency, is taken to be the efficiency of the chain drive system (99%).

Section 5.6: Light-Duty AWD

Recall that all wheel drive vehicles address the problem of 4WD transfer cases not allowing 4WD operation on pavement by replacing the transfer case of 4WD drivelines with a center "differential" which often has a center differential that is similar to the differential in a transaxle. Thus, the following equation presents the efficiency calculation for an all-wheel drive vehicle in terms of its components, the transaxle differential and two conventional RWD differentials (although one of these is the front conventional differential):

$$\eta_{diff,LD\_AWD}=\eta_{diff,LD\_FWD}*\eta_{diff,LD\_RWD}$$

Equation 5.26: Decomposition of Light-Duty All Wheel Drive Differential's Efficiency in Terms of Other Differentials' Efficiencies

The reader will note that only one of the axle differentials is considered. This is due to the theory of equal torque splitting. From the center differential, half of the remaining torque is diverted to each axle differential. Since the axle differentials and their input parameters are equivalent, their efficiencies will be equal as well.

Section 5.7: Heavy-Duty RWD

Inland Truck Parts allowed pictures to be taken of the main gear pair of a heavy-duty rear axle differential. The following equation presents the efficiency calculation with the order of the components considered.

$$\eta_{diff,HD\_RWD}=\eta_{seal,1}*\eta_{bear,1}*\eta_{wind,cyl,1}*\eta_{gp,hyp}*\eta_{wind,sph}*\eta_{wind,cyl,2}*\eta_{bear,2}*\eta_{seal,2}$$

Equation 5.27: Decomposition of Heavy-Duty Rear Wheel Drive Differential's Efficiency into Constituent Component Efficiencies

Section 5.8: Heavy-Duty RWD Dual Differential

It is assumed that this model behaves in much the same way as that of the heavy-duty RWD conventional differential, except that the dual differential has a two-speed gear box attached upstream in the powertrain. Fellow researcher Kyung Jin Kim developed a two-speed gear box model for use as part of those of his manual transmission models that include a secondary two- or three-speed gear box. The calculations for 2-speed gear box performance may be executed at the end of the transmission just as well as at the front of the differential for the purposes of modeling. Mr. Kim was gracious enough to allow one of his gear boxes to be modified to suit the purposes of this HD RWD dual differential model. The following equation presents the efficiency calculation with respect to the other component models:

$$\eta_{diff,HD\_Dual} = \eta_{2speed\ gb} * \eta_{diff,HD\_RWD}$$

Equation 5.28: Decomposition of Heavy-Duty Rear Wheel Drive Dual Differential's Efficiency into Constituent Component Efficiencies

Section 5.9: Heavy-Duty Tandem Axle Conventional and Dual Differential Configurations

The reader should recall the discussion in Section 2.2.7 regarding the live tandem configuration of axles. As will be discussed in this and the next section, the tandem axle configuration for three-axled vehicles may employ either conventional differentials or dual differentials in addition to the center differential. As models similar in nature to these components already exist, the efficiency calculation for the tandem axle configuration with conventional differentials and, indeed, that for the tandem axle configuration with dual differentials may be written in terms of existing efficiency calculations.

$$\eta_{diff,HD\_Tan\_Conv} = \eta_{diff,LD\_FWD} * \eta_{diff,HD\_RWD} ; \eta_{diff,HD\_Tan\_Dual} = \eta_{diff,LD\_FWD} * \eta_{diff,HD\_Dual}$$

Equations 5.29: Decomposition of Heavy-Duty Tandem Axle Conventional and Dual Differential Configurations' Efficiencies in Terms of Other Efficiencies

Section 5.10: Heavy-Duty Tag Axle Configuration

Tag axle configurations were also discussed in Section 2.2.7. As they represent a potential increase in fuel economy, the current trend of increasing regulation on transportation technology would suggest that such configurations will become more mainstream in years to come. As the tag axle configuration is schematically identical to the heavy-duty rear wheel drive conventional or dual configuration save for extra drag on the tag axle, those respective models will be called in the case of an encounter with a tag axle configuration. The contribution of the tag axle to the efficiency of the configuration is neglected in the model.

Chapter 6: Differential Model Code Running Modes

This differential model is written in such a way that it can be used several different ways conveniently. The drive cycle simulator may choose to use the model in one of these three ways to balance concerns with computational power and storage spacely discussed in the following subsections.

Section 6.1: Run Every Time During Simulation

The first method requires the drive cycle simulator to call the selected differential model at each time interval with the instantaneous corrected torque input to the differential (transmission output torque), input rotation speed (transmission output speed), and gear ratio(s). With a long drive cycle, this method will be computationally demanding but requires only temporary data storage for the variables of the differential model while it is running.

Section 6.2: Run Once Before Simulation

The second method requires the drive cycle simulator to call the appropriate differential model once before the drive cycle with arrays of input torque and speed and one gear ratio for each gear pair. This method requires temporary storage, but the linear interpolation algorithm is far less computationally demanding than a single input from the efficiency model. The user will notice an improvement in model performance over the first method once the number of time intervals exceeds the number of elements in the arrays by enough to offset the linear interpolations conducted up to that point.

Section 6.2.1: Generate 2-D Differential Efficiency Map

With knowledge of the gear ratio of the differential used in the simulated vehicle, an efficiency map can be generated which spans the range of input torque and speed.

Section 6.2.2: Interpolate Within Map When Differential Efficiency Needed

When the differential efficiency is needed for a given differential input torque and speed, an interpolation algorithm can be used to extract the needed efficiency from the efficiency map rather than directly from the differential model. The interpolation algorithm is far more efficient than the differential model, but the initial investment of time must still be made.

Section 6.3: Run Once and Discard Code

The third method requires the drive cycle simulator to check for existing differential efficiency maps stored as functions of differential input torque, speed, and gear ratio. If existing maps are not found, the drive cycle simulator will call a differential model as it would in the second method. However, the drive cycle simulator will call the differential model multiple times, each time with a new gear ratio.

Section 6.3.1: Generate 3-D Differential Efficiency Map

In effect, the drive cycle simulator will build a three-dimensional map of differential efficiency with respect to differential input torque, speed, and gear ratio. This will require the most storage of these methods, but once the initial time investment is made into building these maps, the differential models will never need to be called again.

Section 6.3.2: Interpolate Within Map When Differential Efficiency Needed

As a result, linear interpolations of the maps will be the only calculation required for differential efficiency. This third method would be the least computationally demanding method.

Section 6.4: Examples of Inputs/Outputs for Current Differential Models

The same inputs were used for all of the models except for the dual differential, which requires more inputs. The other programs require three inputs:

- The scaled input torque to the differential, which is the scaled output torque from the transmission,

- The differential input shaft speed in rpm, and
- The differential's gear ratio.

For the other models, the following inputs were used:

Table 6.1: Example Scaled Input Torque to the Differential

τ/τ_{ref}	Torque fraction is held constant while speed increases to the right.				
Torque fraction increases at constant speed.	0	0	0	0	0
	0.25	0.25	0.25	0.25	0.25
	0.5	0.5	0.5	0.5	0.5
	0.75	0.75	0.75	0.75	0.75
	1	1	1	1	1

Table 6.2: Example Input Shaft Speed to the Differential, in rpm

Speed is constant while torque varies.	0 rpm	1000 rpm	2000 rpm	3000 rpm	4000 rpm
	0 rpm	1000 rpm	2000 rpm	3000 rpm	4000 rpm
	0 rpm	1000 rpm	2000 rpm	3000 rpm	4000 rpm
	0 rpm	1000 rpm	2000 rpm	3000 rpm	4000 rpm
	0 rpm	1000 rpm	2000 rpm	3000 rpm	4000 rpm
N, or ω	Speed increases to the right while torque fraction is held constant.				

Gear_ratio = 3.

For the dual differential model, the low gear ratio and chosen gear of the two-speed gearbox also need to be set.

Figures 6.1-6.6 illustrate the efficiency values obtained by providing the aforementioned inputs.

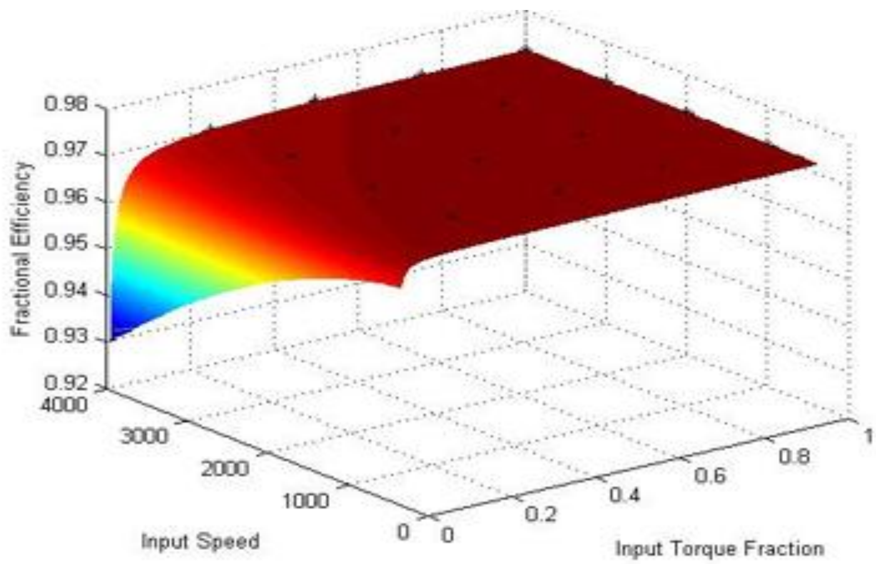


Figure 6.1: Efficiency map of a light-duty front wheel drive differential.

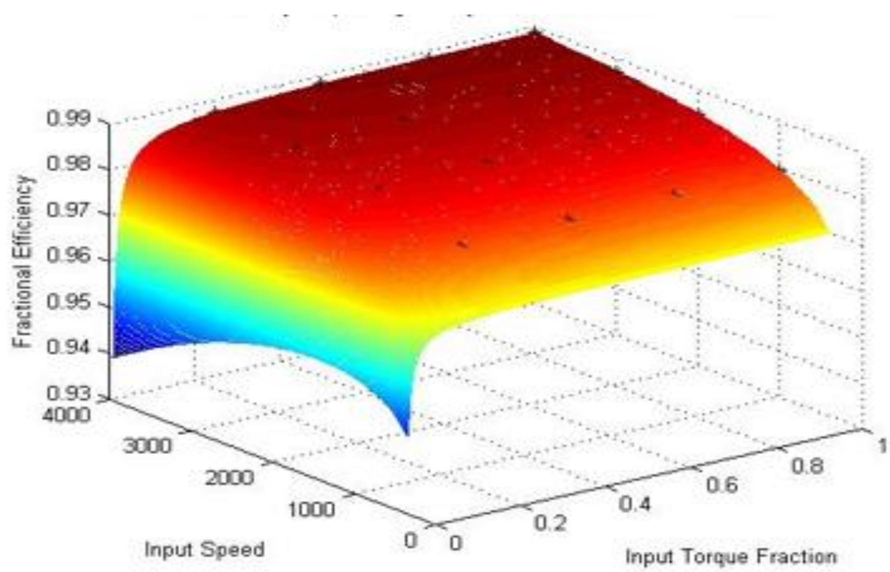


Figure 6.2: Efficiency map of a light-duty rear wheel drive differential.

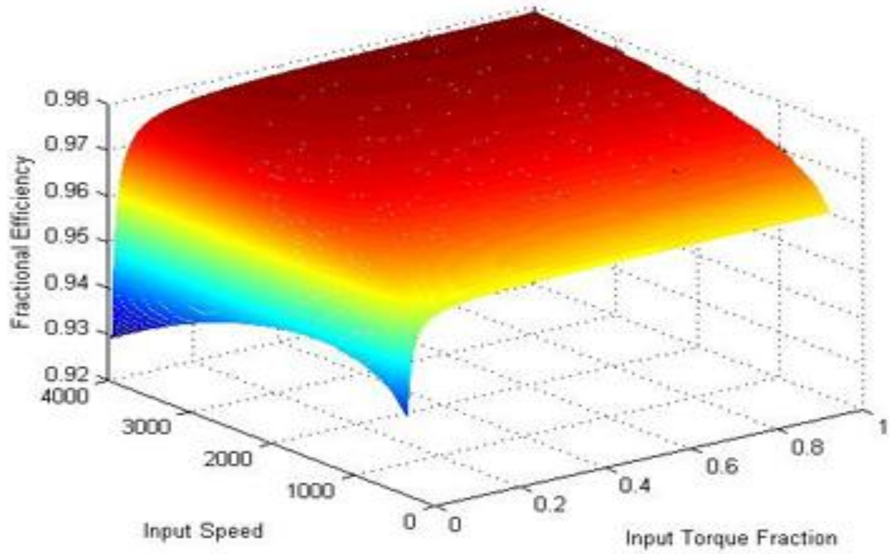


Figure 6.3: Efficiency map of a light-duty four wheel drive differential.

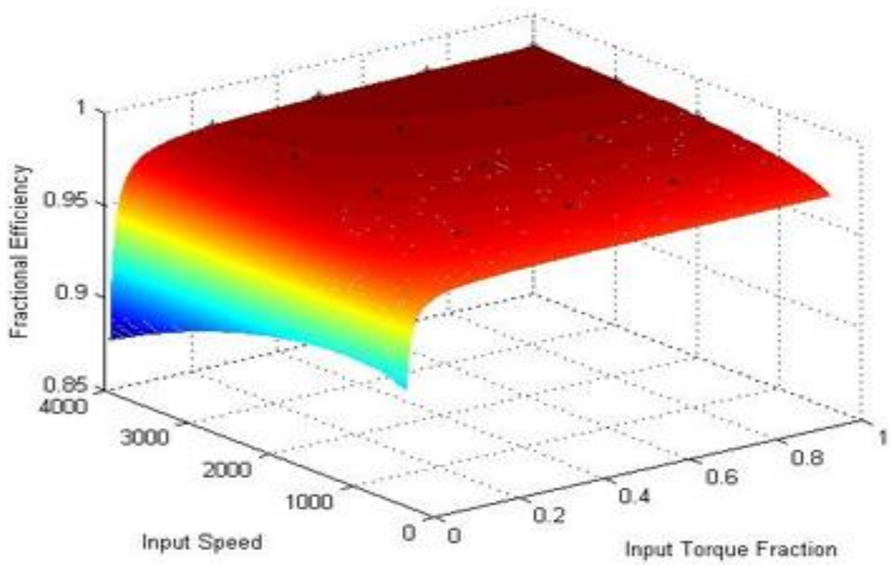


Figure 6.4: Efficiency map of a light-duty all wheel drive differential.

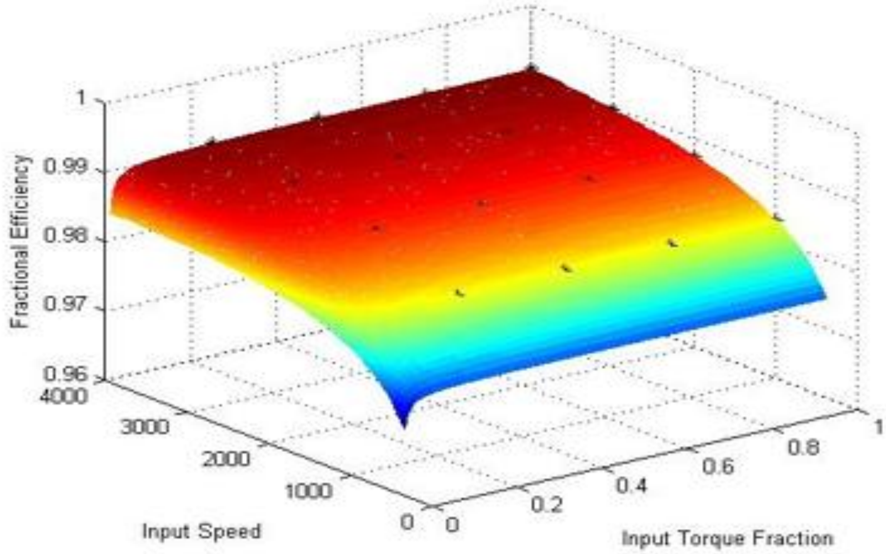


Figure 6.5: Differential efficiency map for a heavy-duty vehicle with a single rear axle or with a tag axle.

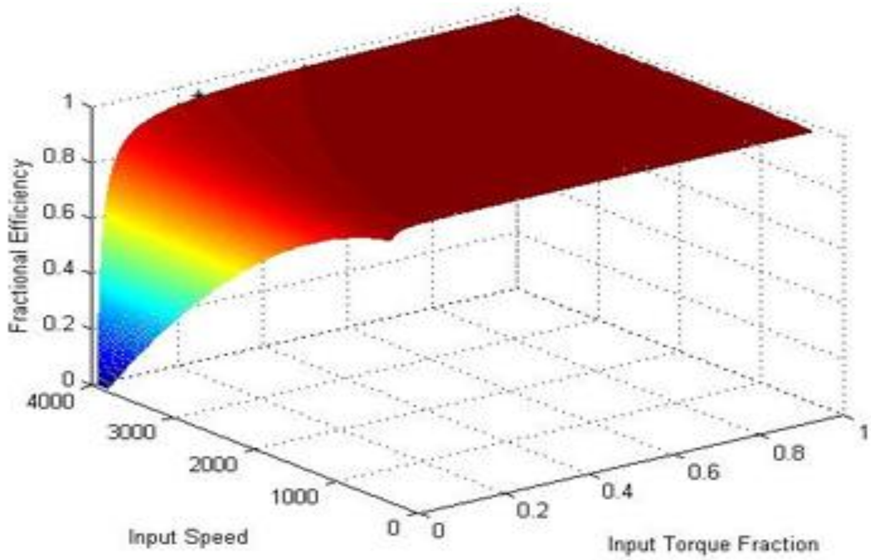


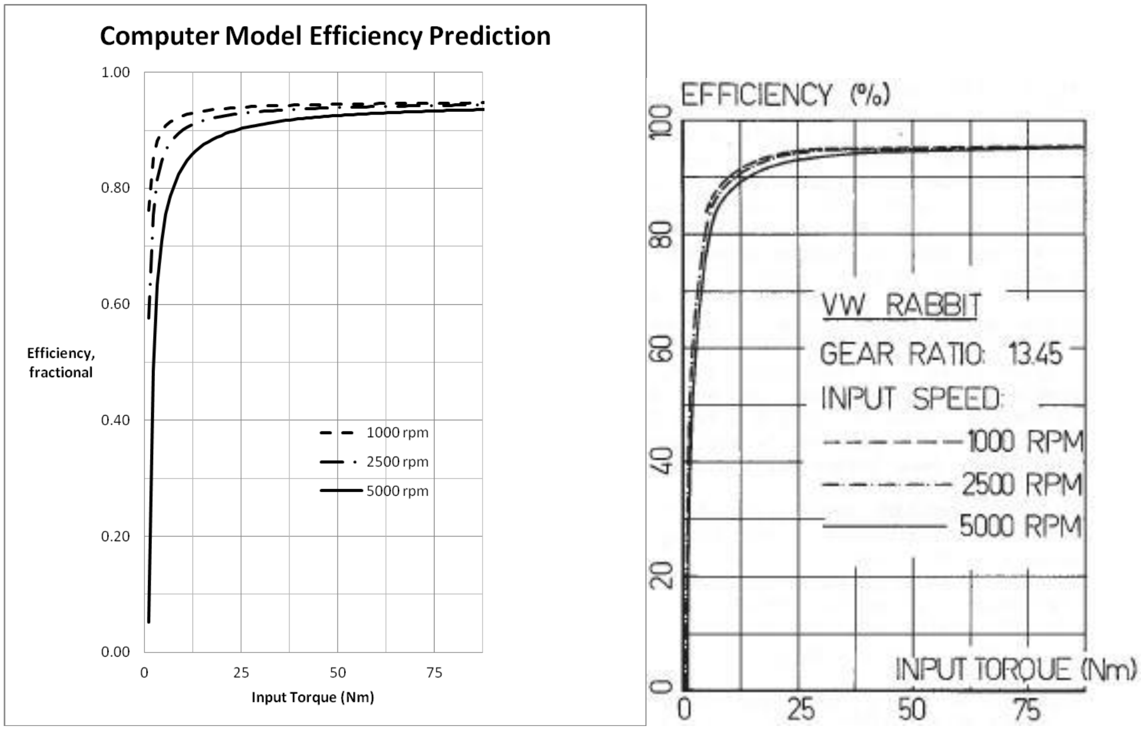
Figure 6.6: Efficiency map for a heavy-duty dual differential for either a single rear axle or with a tag axle.

Chapter 7: Proof of Concept Tests

In this section, the outputs from the models are compared with existing data and other models to determine the validity of the differential efficiency models developed as part of TxDOT Project 0-5974. Data exists for a transaxle, and a curve fit exists for a rear-wheel-drive light-duty vehicle.

Section 7.1: Light-Duty FWD (Manual Transaxle)

SAE Paper 820741 (van Dongen, 1982) contains graphed efficiency data for both manual and automatic transaxles from VW Rabbit vehicles ca. 1980's. This paper documents the efficiency of a ~1980 Volkswagen Rabbit transaxles (FWD transmission + differential) with both a 4-speed manual and 3-speed automatic transmission. Since a transaxle assembly is a transmission and differential combined, co-investigator Kyung Jin Kim provided a model for each transmission. The following figures, part of a proof of concept test that considered all of the gears of the transmission, demonstrates the ability of the combined transmission and differential models to predict the efficiency of a light-duty front wheel drive vehicle. Data were excerpted from Figure 6.1, combined with data from co-investigator Kyung Jin Kim's manual transmission model, and scaled appropriately to be compared side by side with the published data.



Figures 7.1A and 7.1B: Combined Transmission and Differential Efficiency Model Outputs Compared with Available Data (van Dongen, 1982) for a Manual Transaxle

Section 7.1.1: Lubricant Concern

An investigation into this situation ruled out a couple of effects. As mentioned before, even by assuming the efficiency of the differential to be unity, the efficiencies of the data were unattainable. The temperature of the lubricant also had an effect on the efficiencies. The operating temperature of the transmission and differential were presumed to be 50 degrees Celsius above ambient, which was assumed to be near 30 degrees Celsius. However, as with neglecting the differential efficiency, the operating temperature effects could not resolve the issues with the higher gears without compromising the fit from the lower gears.

The discrepancy between the models and the data may lie in the architecture differences between the transmissions. The highest gear of a manual transmission typically has a noticeably higher efficiency than the other gears. Hence, the fourth gear data from the transaxle would likely correspond better with the fifth gear of the manual transmission model. Even so, a five-speed transmission simply must have more moving parts than a four-speed transmission. As a result, for transmissions of equally sophisticated design, a four-speed transmission will simply be more efficient than a five-speed transmission.

Section 7.2: Light-Duty RWD

Matthews (2011) presents a curve fit to data for the differentials of light-duty rear wheel-drive vehicles:

$$\eta_{diff,LD\_RWD} = \begin{cases} 0.6652+3.732*10^{-3}*S-1.061*10^{-5}*S^2 & ; S \leq 150 \text{ kph} \\ 0.987 & ; S>150 \text{ kph} \end{cases} \tag{C.19}$$

This equation relates the efficiency of a light-duty rear-wheel-drive differential to the speed of the vehicle (S) in which it is installed. It is assumed that the curve fit data were collected in tests where vehicle speed was held constant. Extracting coastdown coefficients from the Environmental Protection Agency website for several light-duty rear-wheel-drive vehicles (see Subsection C.7.3.2) allows the estimation of the road load forces acting on the car as a function of vehicle speed. These road load forces must be counteracted by the torque applied by the powertrain on the wheels by acting over the moment-arm that is the rolling radius of the tire. Tire data were collected from the same EPA source for the same vehicles as the vehicles associated with the coastdown coefficients. The rolling radius was assumed to be constant, introducing a slight error in the calculation of driving force from the driving torque. Nevertheless, obtaining coastdown coefficients and tire radii allowed the estimation of differential output torque required to maintain constant vehicle speed under road load conditions.

Using the curve fit for efficiency (Equation C.19) and the calculated required differential output torque allowed for approximation of the differential input torque for the model. Since the source of the differential output torque (via the coastdown coefficients) is more reliable (EPA) than the source of the curve fit, an iterative process was used to find the predicted differential efficiency while matching the differential output torque obtained from the coastdown coefficients. In order to accomplish this, the peak torque available from each of the vehicles was estimated.

Table 7.1: Estimated Peak Torque Available from Vehicles in EPA Coastdown Coefficient Tests

Vehicle	Peak Engine Torque (N.m)
Ford F-150	400
Mercedes 300E	219.6
Cadillac DeVille	372.8
Nissan 240SX	206.1
Dodge Caravan	227.8
Chevrolet Lumina	250.8
Honda Civic	157.3

Once the iterative process reached completion, the model's efficiency predictions were compared with the curve fit.

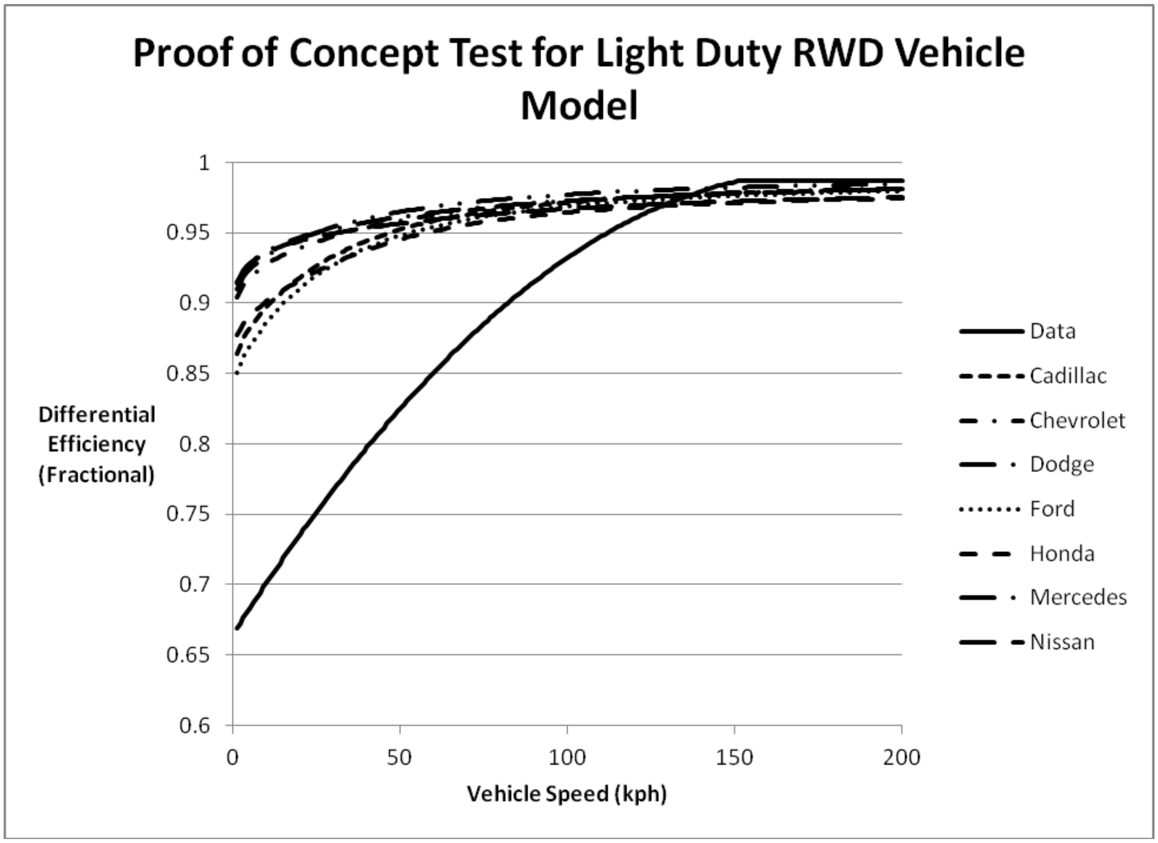


Figure 7.2. Differential efficiency as a function of speed from available data (Matthews 2011) compared to the present model predictions for a light-duty rear wheel drive vehicle. Details regarding the experimental vehicle(s) were not available.

The curve labeled Data is clearly separated from the rest of the curves, especially at low vehicle speeds. This Data curve represents the curve fit provided in Matthews' appendix. The other data curves represent the model's efficiency predictions of each of the vehicles. Since the fit is so poor across the entire selection of vehicles from Cadillac to Nissan, the curve fit and the EPA information were used to compute the approximate torque losses across the differential, removing the uncertainty of the model. The resulting graph is confounding.

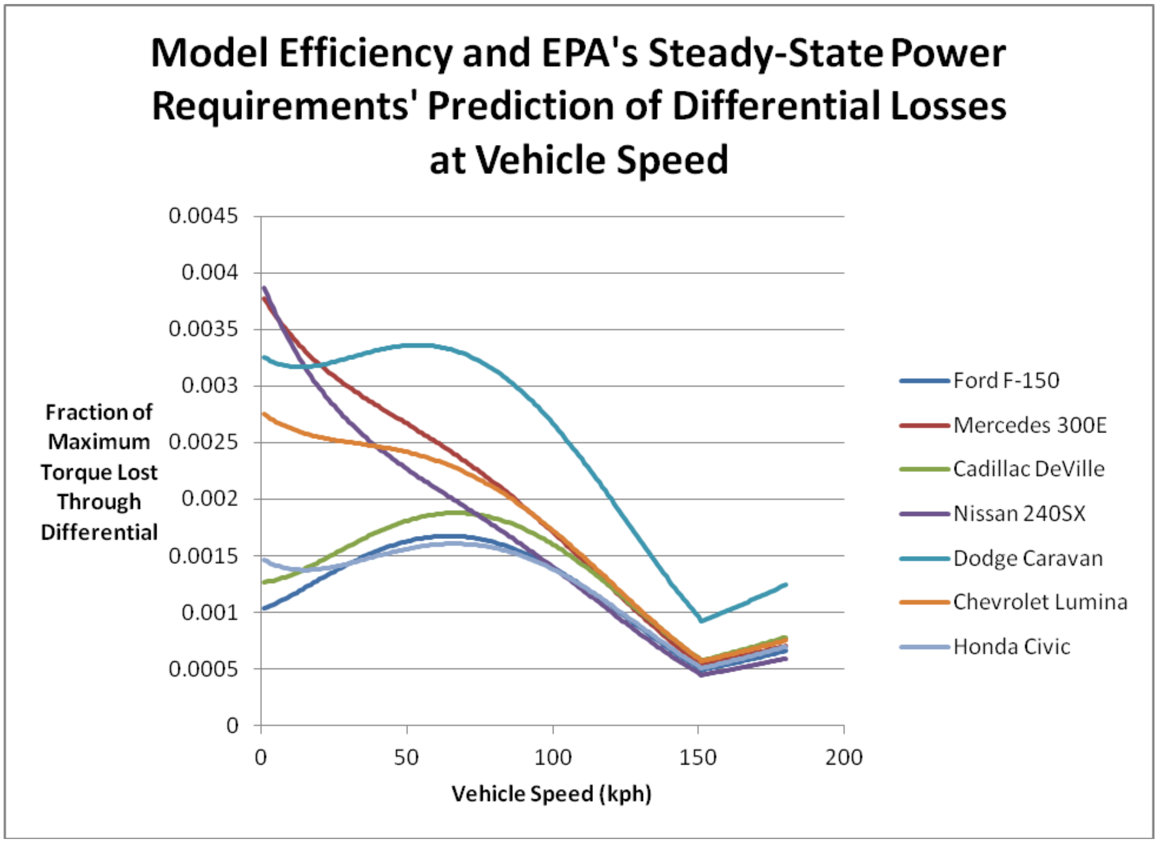


Figure 7.3. Calculated torque lost via the RWD differential as a function of speed for a light-duty rear wheel drive vehicle. Details regarding the experimental vehicle(s) were not available.

According to the EPA data and the curve fit, torque losses should be higher at low speed and high speed than at approximately highway speeds. The mechanisms for increased torque loss at high speed are windage and, to a lesser extent, bearings. One mechanism is available for increasing the torque loss at low speed, and that is the hypoid gear friction. The composite coefficient of friction is higher at low speed than at high speed for hypoid gear pairs. However, published friction coefficient data for hypoid gears are not sufficiently high at low speed to accommodate the increased torque losses predicted at low speed for light-duty rear-wheel-drive vehicles.

This calls into question the curve fit and EPA's coastdown coefficients. However, the lack of other available data prevents further model validation. It is interesting to note that the trends in the curve fit are mimicked slightly by the model. Furthermore, the model represents a vast improvement over the assumption of constant efficiency. Ultimately, however, the model could use improvements in most of its aspects.

Section 7.2.1. Tire Rolling Radius Model

The tire rolling radius was assumed to be constant at all speeds and directly dictated by the specifications of the tire. Tire manufacturers express tire size in one of a couple of different ways, but the tire diameter is always attainable.

Section 7.2.2. Coastdown Coefficients and Tire Radii from EPA

The data in Table 7.1 were extracted from Reineman and Nash (1995), a report by from the US EPA National Vehicle and Fuel Emissions Laboratory for the Dynamometer Comparison Study Task Force.

Table 7.1. Coastdown coefficients and tire sizes used in the simulations for light-duty RWD vehicles.

Vehicle	A-Coefficient	B-Coefficient	C-Coefficient	Tire Size
Ford F-150	21.8	0.9315	0.03266	P235/75R15
Mercedes 300E	40.4	0.3529	0.01636	195/65VR16
Cadillac DeVille	15.9	0.4716	0.02444	P205/70R15
Nissan 240SX	48.1	0.0096	0.01809	195/60R15
Dodge Caravan	27.1	0.4804	0.02494	P205/70R15
Chevrolet Lumina	26.9	0.4216	0.01637	P215/60R16
Honda Civic	15.4	0.1384	0.01960	P175/70R13

Chapter 8: Summary and Conclusions

The basis for a model for the efficiency of an automotive differential has been provided. The results from proof of concept tests were analyzed to determine the validity of the model.

While the proof of concept tests seem to demonstrate that the model leaves the user wanting for accuracy, the reader is reminded that the fundamental model could very well have been a step back. Instead, the fundamental model is, in both cases of proof of concept, an improvement over the assumption of constant efficiency. Further, no extra data collection aside from what is already available was necessary to create this code. Finally, further improvements to the model could bring the model's results much closer to the accepted values.

Chapter 9: Recommendations for Future Work

This section has been left for topics of interest which were not incorporated due to any of several mitigating factors. For the most part, their effects on the efficiency of a differential will have been secondary in nature to the topics that were considered.

Section 9.1: Bearing model update

The current model comes from SKF information available in 1999 to Khonsari and Booser. Over the past decade, bearings may have improved.

Section 9.2: Seal model update

The current model neglects to consider many effects that would cause the seal to behave differently in real life than as modeled. In addition to material selection and seal pre-load, the seal model itself needs an improved tribological basis that covers the interaction of seal materials with shaft materials and differential lubricants. This basis should tease out some speed and torque dependences in the efficiency of a seal in much the same way that the entire differential model teases out the effects of the same parameters in the efficiency of the entire differential.

Section 9.2.1: Material

One common seal material is nitrile, though many others may exist.

Section 9.2.2: Pressure

The pre-load on a seal will determine the normal force on each surface. Not only will this affect the frictional force of the seal, but this will also affect the lubrication regime of the seal. Pressure is one of the components of the Stribeck number.

Section 9.3: Gear pair model updates

The gear pair models are based in part on a reference that is over 50 years old. In the meantime, researchers have been investigating the effects of various parameters on the efficiency of these gear pairs, parameters that were neglected by the old reference.

Section 9.3.1: FEA

Finite element analysis of the gear pair would allow for a better approximation of tooth contact stress and area. These quantities could be used in a tribological model to create a more accurate friction loss model.

Section 9.3.2: Friction

The coefficient of friction for the gear pair efficiency was taken from tabulated data or assumed to be constant. However, an examination of tribology will reveal the existence of a Stribeck curve. Its application to gear pair efficiency would serve to further improve the accuracy of a gear pair efficiency model, as it would account for the change in the coefficient of friction with respect to rotational speed of the gears.

Section 9.3.3: DNS

Instead of considering the Stribeck curve, gear tooth profiles and roughness information could be used to simulate gear pair friction directly. Coupled with asperity contact models, direct numerical simulation would allow the fluid between the gear teeth to be modeled and the shear stress extracted at any location and time. With sufficient resolution, direct numerical simulation could predict the gear pair friction with better accuracy than existing models.

Section 9.4: Windage loss model updates (DNS)

The windage models are simplified by assumptions that are not necessary. Now that direct numerical simulation techniques are available, equations that were once daunting now have numerical solutions readily available.

Section 9.5: More proof of concept tests

The proof of concept tests that have already been done serve to confirm the accuracy of the helical and hypoid gear pair models to some degree. Since uncertainty exists with the hypoid gear pair model from the light-duty RWD proof of concept test, proof of concept tests from other powertrains that use hypoid gear pairs would be desirable to increase confidence in the model.

Section 9.6: Asymmetry consideration

The assumption that all powered wheels are receiving the same torque at all times may not be valid. Tires alone are compliant enough to allow small oscillations to exist between the half-shafts, which would cause the differential to execute some of its more complicated motions in a similarly oscillatory manner. Furthermore, the motion of each half-shaft may occur under greater load, affecting the efficiency calculation.
In other respects, powertrains were sometimes assumed to be symmetric where it is known that a symmetry does not exist. Resolving these asymmetries while requiring that all wheels receive the same torque would have required iteration and other sophisticated coding techniques for a marginal improvement to the model.

Section 9.6.1: Bearing loading

Right now, bearings are presumed to be spaced equidistant from the application of force on a rotating component. As a result, both bearings carry half of that applied force. As one bearing begins to take on more of the load, its efficiency may diminish faster than the efficiency of the other bearing increases.

Section 9.7: Turning and loss of traction effect simulation

As the vehicle travels through a turn, the differential will execute a more complex motion. This complex motion would diminish the efficiency of the differential. In addition to desiring to know the duration of the turn, some information should also be known regarding the severity of the turn. For instance, the effective radius of the turn would indicate the difference in speed between the wheels of a given axle.

Section 9.8: Ambient temperature effects

The temperature difference between the operating temperature of the differential and the ambient temperature remains roughly constant for normal operation of the differential. As the ambient temperature changes from standard conditions, the operating temperature of the differential will change. This will impact the viscosity of the lubricant of the differential. Seasonal variations, at least, could be captured and analyzed to determine whether or not the difference in lubricant viscosity has any effect on the efficiency.

Section 9.9: Unexplored and future differential types

Just as the term differential has come to mean many things over the last few decades, the term should be expected to cover an ever-expanding variety of mechanisms and gadgets in the future. In order to stay applicable, the differential model will require updates.

Section 9.9.1: Mining equipment

Mining equipment can be larger than the largest heavy duty vehicle that transports regular cargo. Their rougher terrain and other demands may necessitate a different powertrain configuration. It may be worth investigating whether or not this differential model is applicable to that equipment.

Section 9.9.2: Military applications

The military may have vehicles for special applications that require that the powertrain be configured in a way not explored in this thesis.

Section 9.10: Limited slip device parasitic loss model

All limited slip devices were neglected in the model. However, one of the common types of slip limiters is a series of plates rotating in an enclosure. Depending on the size of the plates, the efficiency of the limited slip, even while inactive, may approach the gear pair windage efficiency.

Section 9.11: Unconfined aerodynamic shaft windage loss model

Shafts exposed to outside air were neglected in the model. However, a flat plate assumption could be used to estimate the windage losses of air on an unconfined rotating shaft.

Section 9.12: Chain drive model

Right now, the chain drive model for the transfer case assumes that the transfer case is 99% efficient. A cursory literature search may reveal a better approximation of the efficiency of a chain drive.

Section 9.13: Universal and Constant-velocity Shaft Joint Models

Both shaft joint types were neglected in the model. A cursory literature search may reveal an existing approximation of the efficiencies of these components.

Section 9.14: Compensation for vehicle age and wear

As components age and wear out, their efficiency may decrease. A refined literature search may reveal some information on a way to approximate the effects that age and wear may have on the efficiency of a differential.

Section 9.15: Vibration model

The engine produces a cyclic load on the powertrain. As a result, vibrations are encouraged. Vibrations could cause the components of a vehicle to dissipate mechanical energy to heat in a mechanism not yet explored. A cursory literature search may reveal an existing approximation for the losses associated with vibrations.

Appendices

Appendix A: Flowcharts

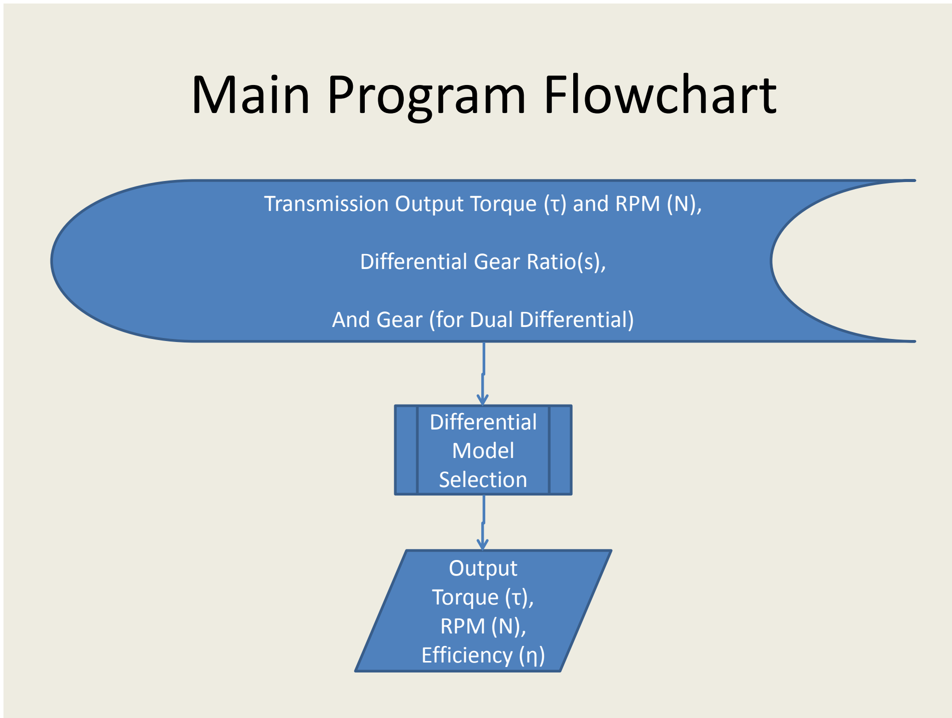


Figure A.1: Main Concept of Models of Efficiency of Differentials

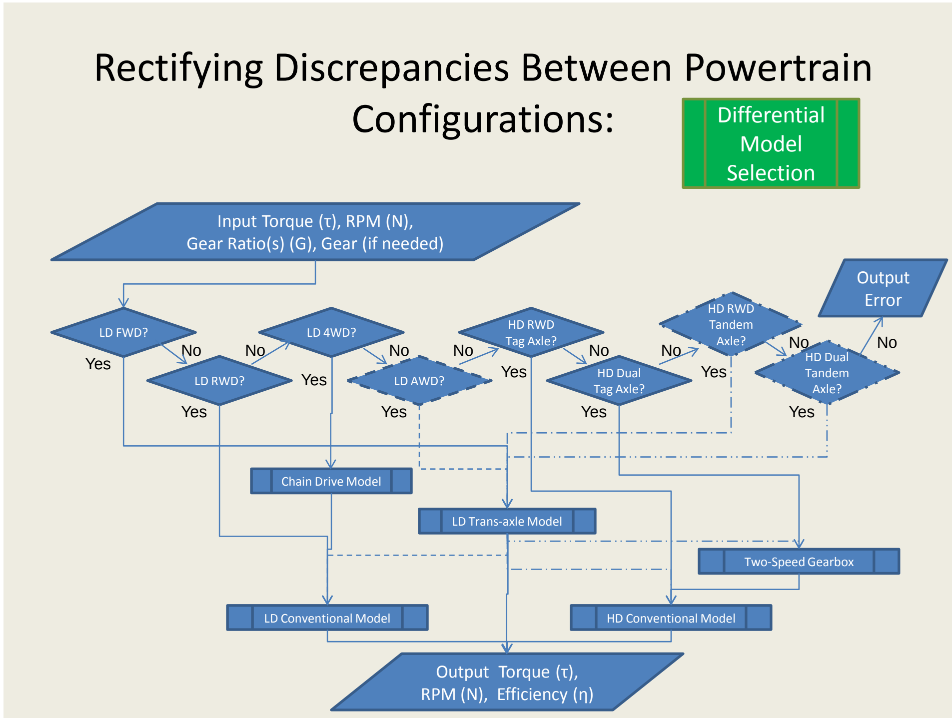


Figure A.2: Subroutine Order for and Sharing between Powertrain Configuration Options

Appendix B: Code

The programs used in modeling the differentials are reproduced in this appendix to ensure repeatability of results. In addition, owing to the somewhat modular nature of the programs, improvements can be made to the presented programs with minimal hassle to enhance the accuracy of the results.

Section B.1: Light-Duty Transaxle Model

```
function [efficiency,torque,speed] = EffDiffTrans(scaled_torque,speed,gr)
%EffDiffTrans Efficiency of transaxle predicted from 2005 Equinox data
%   Inputting the normalized torque scaled against the torque capacity
%   of the differential of the vehicle in question and axle rotating
%   speed will yield an efficiency.  Axle rotating speed (rpm) is a
%   misnomer: it corresponds to transmission output rotating speed.
%
%   Written by James Vaughn
%   Last edited 31 July 2011

[a,b]=size(scaled_torque);
efficiency = ones(a,b);
equinox_max_torque=285*4.69*3;
% Obtained the GM 60-Degree V6 Engine peak torque (in N-m) and the Aisin AF33
% Transmission 1st gear ratio, both from Wikipedia.  Also included a torque
% ratio of 3 for a torque converter, an artifact of automatic transmissions.

torque = scaled_torque.*equinox_max_torque;   % Defined
equinox_G=2.652173913043478260869565217391 3;
% Measured from the 2005 Chevy Equinox

%   Gear pair windage and loss of the 2005 Chevy Equinox; needs to be
%   revised.

%       Pinion windage

[rho_air,mu_air,nu_air]=walthersEqn(0);
```

```
% Properties of air at operating temperature and other conditions
R_p_pinion=0.12794339139344262295081967213115;
% Measured from the 2005 Chevy Equinox
L_fw=0.034925;
% Measured from the 2005 Chevy Equinox
OD_D=0.25985532786885245901639344262 3;
% Measured from the 2005 Chevy Equinox
[rho_oil,mu_oil,nu_oil]=walthersEqn(1);
% Properties of ATF at operating temperature and other conditions
mu=mu_oil;
nu=nu_oil;
C=0.46727151639344262295081967213115;
% Measured from the 2005 Chevy Equinox
alpha=0.34906585039886591538473815369772;
% Measured from the 2005 Chevy Equinox
gamma=0.48136528445852719055370932643102;
% Measured from the 2005 Chevy Equinox
t_D=23;
% Measured from the 2005 Chevy Equinox
G=gr;
if gr<equinox_G
   t_D=t_D*equinox_G/gr;
   OD_D=OD_D*equinox_G/gr;
   R_p_pinion=R_p_pinion*equinox_G/gr;
   C=C*(1+1/gr)/(1+1/equinox_G);
else
   C=C*(1+equinox_G^2/gr)/(1+equinox_G);
end
mu_f_gp=0.04;
```

```
% Average of tabulated values from Dudley's Gear Handbook Fig. 12.1 and Dr.
% Matthews' appendix on efficiency of a transmission
X=0;                        % Pinion being lubricated by the driven gear
C_E=0.7;                    % Corresponds to loose enclosure (SOURCE?)
torque_pinion = 1.069e-4 .* C_E .* (1+2.3*L_fw/R_p_pinion) .* speed.^1.85 .*
OD_D^4.7 .* (X*(rho_oil*nu_oil^.15)+(1-X)*(rho_air*nu_air^.15));   eta_p_windage =
1 - torque_pinion./torque;
torque = torque .* eta_p_windage;
efficiency = efficiency .* eta_p_windage;
[efficiency,torque] = EtaNTorqCorrect(efficiency,torque,true);

%       Gear pair friction

epsilon = 1.1337;     % Calculation needed to be bypassed for G=1.
eta_gear = 1 - ((mu_f_gp / (2*C/(1+G)*cos(alpha))) * abs(1-1/G) * (C*sin(alpha)/(1+G))
* (1+G^2)/abs(1-G) * (1-epsilon+epsilon^2));
torque = torque .* G .* eta_gear;
speed = speed ./ G;
efficiency = efficiency .* eta_gear;
[efficiency,torque] = EtaNTorqCorrect(efficiency,torque,true);

%       Driven gear windage

X=0;                   % Corresponds to completely dry immersion fraction
R_i_gear=0.06985;      % Measured from the 2005 Chevy Equinox
if gr>equinox_G
   R_i_gear = R_i_gear*gr/equinox_G;
end
```

```
torque_gear = 1.069e-4 .* C_E .* (1+2.3*L_fw/R_p_pinion/G) .* speed.^1.85 .*
((OD_D*G)^4.7-(2*R_i_gear*G)^4.7) .* (X*(rho_oil*nu_oil^.15)+(1-
X)*(rho_air*nu_air^.15));
eta_g_windage = 1 - torque_gear./torque;
torque = torque .* eta_g_windage;
efficiency = efficiency .* eta_g_windage;
[efficiency,torque] = EtaNTorqCorrect(efficiency,torque,true);

%   Spherical windage of the 2005 Chevy Equinox

r=0.06985       % Measured from the 2005 Chevy Equinox
R=0.0762        % Measured from the 2005 Chevy Equinox
if gr>equinox_G
   r=r*gr/equinox_G;
   R=R*gr/equinox_G;
end
eta_sphere = 1 - (4.*pi().^2./45.*mu_air.*speed.*r.^4./(R-r)) ./ torque;
% Derived using calculus
efficiency = efficiency .* eta_sphere;
% Derived from first principles
torque = torque .* eta_sphere ./2;
% Derived from first principles
[efficiency,torque] = EtaNTorqCorrect(efficiency,torque,true);

%   Bearing losses of the 2005 Chevy Equinox

lub_fric_fac_index=3;            % Defined for oil bath lubrication
mu=0.0018;
lub_fric_fac=[4.2,6,8,9];
```

```
f_1=20;
% These values of data were obtained from Khonsari and Booser's Applied
% Tribology, 2nd ed.
f_2=25;                         % Tables 15.1, 2, and 3.
f_L=lub_fric_fac(lub_fric_fac_index);
R_p=0.339328125;                % Measured from the 2005 Chevy Equinox
numBearings=2;
P = torque.*R_p./cos(pi()/9)./numBearings;
% Load on the bearings, derived from first principles
d=68.150711090682830570592951793237;
% Measured from the 2005 Chevy Equinox (mm)
D=80.850711090682830570592951793237;
% Measured from the 2005 Chevy Equinox (mm)
d_m=0.5*(d+D);                  % Used in M_L
M_P=5e-4*mu*P*d;
% Equation 15.1, page 462 from Khonsari and Booser's Applied Tribology, 2nd
% ed.

[a,b]=size(efficiency);
% This block is required for the possibility of an array of efficiencies.
M_L=zeros(a,b);                 % Pre-allocation for speed
for c=1:1:a
    for e=1:1:b
        if(2*pi()/60*speed(c,e)*nu>2000)
            M_L(c,e)=1e-10*f_L*(2*pi()/60*speed(c,e)*nu)^(2/3)*d_m^3;
            % Equation 15.3, page 463 from Khonsari and Booser's Applied
            % Tribology, 2nd ed.
        else
            M_L(c,e)=1.6e-8*f_L*d_m^3;
```

```
            % Equation 15.4, page 463 from Khonsari and Booser's Applied
            % Tribology, 2nd ed.
        end
    end
end

M_S=1e-3*(((d+D)/f_1)^2+f_2);
% Equation 15.5, page 464 from Khonsari and Booser's Applied Tribology, 2nd
% ed.
M = M_P+M_L+M_S;
% Equation 15.2, page 463 from Khonsari and Booser's Applied Tribology, 2nd
%ed.
eta_bearing = 1-M./torque;
efficiency = efficiency.*eta_bearing;   % Derived from first principles
torque = torque.*eta_bearing;           % Derived from first principles
[efficiency,torque] = EtaNTorqCorrect(efficiency,torque,true);

%   First cylindrical windage of the 2005 Chevy Equinox

omega = 2*pi()/60*speed;
D=0.047625;                   % Measured from the 2005 Chevy Equinox
a=(0.060325-D)/2;             % Measured from the 2005 Chevy Equinox
L=0.0254;                     % Measured from the 2005 Chevy Equinox
eta_windage = 1-(pi()/4*mu_air*D^3*L/a*omega)./torque;
% Example Problem 8.2, page 320 from Fox, McDonald, and Pritchard's
% Introduction to Fluid Mechanics
efficiency = efficiency.*eta_windage;       % Derived from first principles
torque = torque.*eta_windage;               % Derived from first principles
[efficiency,torque] = EtaNTorqCorrect(efficiency,torque,true);
```

```
%   Seal losses of the 2005 Chevy Equinox

r_shaft=0.01465419138518626304091997251250245;
% Measured from the 2005 Chevy Equinox
F_tan_prime=50+170*0.5;
torque_seal = 2*pi()*r_shaft^2*F_tan_prime;              % Specified in N-m
eta_seal = 1 - torque_seal./torque;
torque = torque.*eta_seal;
efficiency = efficiency.*eta_seal;
[efficiency,torque] = EtaNTorqCorrect(efficiency,torque,true);

%   Second cylindrical windage of the 2005 Chevy Equinox

omega = 2*pi()/60*speed;
D=0.02930838277037252608183994502 5049;
% Measured from the 2005 Chevy Equinox
a=(0.0547083827703725260818399450 25049-D)/2;
% Measured from the 2005 Chevy Equinox
L=0.3048;
% Estimated from the 2005 Chevy Equinox
eta_windage = 1-(pi()/4*mu_air*D^3*L/a*omega)./torque;
% Example Problem 8.2, page 320 from Fox, McDonald, and Pritchard's
% Introduction to Fluid Mechanics
efficiency = efficiency.*eta_windage;        % Derived from first principles
torque = torque.*eta_windage;                % Derived from first principles
[efficiency,torque] = EtaNTorqCorrect(efficiency,torque,true);

%   Renormalization of torque
```

```
torque=torque./equinox_max_torque./G.*2;
[efficiency,torque] = EtaNTorqCorrect(efficiency,torque,false);

end
```

Section B.2: Light-Duty Conventional Differential Model

```
function [ eta,torque,rpm ] = EffDiffRearLD( torque,rpm,g_r )
%EFFDIFFREARLD Efficiency of differential of rear axle 2WD vehicle
%
%   Written by James Vaughn as part of a thesis on behalf of the Texas
%   Department of Transportation, Professor Ronald Matthews, and the VCOST
%   and fuel economy models.  Last edited 31 July 2011.
%
%   Inputs:
%   torque - array of fractional torque from transmission output.
%           quotient of transmission output torque divided by peak
%           engine torque and highest possible transmission gear ratio.
%   rpm - array of rotational speed of transmission output shaft, in rpm.
%   g_r - gear ratio (>=1, please.)
%
%   Outputs:
%   eta - array of efficiency of differential, fractional quantity.
%   torque - array of fractional torque at differential outputs.
%           quotient of differential output torque divided by peak
%           engine torque, highest possible transmission gear ratio,
%           and differential gear ratio.
%   rpm - array of rotational speed of differential output shafts, in rpm.
%
%   Note: requires access to walthersEqn.m.

% INITIALIZE PARAMETERS
```

```
[a,b]=size(torque);
% This block is required for the possibility of an array of efficiencies.
eta = ones(a,b);
peak_torque = 6266.72614;
% 4622.1 ft-lbf converted to N-m, based on 6.2L 1500-series engine,
% approximately 434 ft-lbf * 3 * 3.55
% (torque converter * first gear transmission gear ratio)
%peak_torque = 3000*torquepct;   % Used June 9 as part of POC test.
torque = torque.*peak_torque;
[~,mu_air,~]=walthersEqn(0);
% Properties of air at operating temperature and other conditions;
% unnecessary parameters suppressed.
[~,mu_oil,~]=walthersEqn(3);
% Properties of 80W90 gear oil at operating temperature and other conditions;
% unnecessary parameters suppressed.
r_o_p=(1.9375+2.84375)/4;
% From 1500-series pickup, measured at Inland Truck Parts, in inches
r_p_p=r_o_p-0.1875;
% From 1500-series pickup, measured at Inland Truck Parts, in inches
phi_n=20*pi()/180;
% Assumed that the normal pressure angle is near 20 degrees
G=41/12;
% For 1500-series, at Inland Truck Parts
if g_r<G
    r_o_p = r_o_p * G / g_r;
    r_p_p = r_p_p * G / g_r;
end
```

```
% SEAL


% input seal parameters based on Dr. Matthews' model for seals in Appendix
% C
sealstr=1;
r_shaft = 0.028575;             % Shaft radius, in meters (1.125 inch radius) for heavy-
duty vehicles
F_tan_prime = 50+170*sealstr;         % In N/m

% calculate seal efficiency based on Dr. Matthews' model for seals in
% Appendix C
torque_seal = 2*pi()*r_shaft^2*F_tan_prime;    % Specified in N-m
eta_seal1 = 1 - torque_seal./torque;

% calculate new torque and eta.
torque = torque.*eta_seal1;
eta = eta.*eta_seal1;
[eta,torque] = EtaNTorqCorrect(eta,torque,true);



% BEARING


lub_fric_fac_index=3;             % Defined for oil bath lubrication
nu = 36.6e-6;
% m^2/s, for gear oil at T = 70 degC, according to
% http://www.lloydminsterheavyoil.com/transportscience.htm
```

```
mu=0.0018;                              % These values of data
lub_fric_fac=[4.2,6,8,9];               % were obtained from
f_1=20;                                 % Khonsari and Booser's
% Applied Tribology, 2nd ed.
f_2=25;                                 % Tables 15.1, 2, and 3.
f_L=lub_fric_fac(lub_fric_fac_index);   % Defined
R_p=r_p_p;                              % Defined
numBearings=1;                          % Defined
P = torque.*R_p./cos(phi_n)./numBearings;
% Load on the bearings, derived loosely from first principles
d=r_shaft*2e3;                          % Unit conversion (mm)
D=81/68*d;          % Assumption based on 2005 Chevy Equinox bearings (mm)
d_m=0.5*(d+D);                          % Defined; used in M_L
M_P=5e-4*mu*P*d;
% Equation 15.1, page 462 from Khonsari and Booser's Applied Tribology, 2nd
% ed.

M_L=zeros(a,b);
% Pre-allocation for speed; a and b were defined at the beginning.
for c=1:1:a
    for e=1:1:b
        if(2*pi()/60*rpm(c,e)*nu>2000)
            M_L(c,e)=1e-10*f_L*(2*pi()/60*rpm(c,e)*nu)^(2/3)*d_m^3;
            % Equation 15.3, page 463 from Khonsari and Booser's Applied
            % Tribology, 2nd ed.
        else
            M_L(c,e)=1.6e-8*f_L*d_m^3;
            % Equation 15.4, page 463 from Khonsari and Booser's Applied
            % Tribology, 2nd ed.
```

```
        end
    end
end

M_S=1e-3*(((d+D)/f_1)^2+f_2);
% Equation 15.5, page 464 from Khonsari and Booser's Applied Tribology, 2nd
% ed.
M = M_P+M_L+M_S;
% Equation 15.2, page 463 from Khonsari and Booser's Applied Tribology, 2nd
% ed.
eta_bearing1 = 1-M./torque;             % Defined
eta = eta.*eta_bearing1;                % Derived from first principles
torque = torque.*eta_bearing1;          % Derived from first principles
[eta,torque] = EtaNTorqCorrect(eta,torque,true);



% CYLINDRICAL WINDAGE


% Specifies angular velocity in rad/s from rpm in min^-1.
omega = 2.*pi()./60.*rpm;

% Gathers other required information
% d recycled from above, converted to meters.
d = d/1e3;
D = D/1e3;
a = (D-d)/2;
% Computes clearance from previous inputs, recycling information from above.
```

```
L = 0.1016;
% 4 inches converted to meters, estimated from pictures taken at Inland Truck
% Parts.
mu = mu_air;                    % Can be air or oil lubricant

% Calculates windage efficiency from torque decrement
eta_windage1 = 1-(pi()./4.*mu.*d.^3.*L./a.*omega)./torque;
% Example Problem 8.2, page 320 from Fox, McDonald, and Pritchard's
% Introduction to Fluid Mechanics

% Adjusts values of interest
eta = eta.*eta_windage1;
torque = torque.*eta_windage1;
[eta,torque] = EtaNTorqCorrect(eta,torque,true);



% HYPOID GEAR PAIR



% First, the windage on the upstream pinion
% Windage calculation based on bevel gear windage calculation available
% from Dudley's Gear Handbook, 2nd ed.

% Psi's are helix angles of the pinion and gear and are estimated by
% geometric relationships.
l_fw_p=2.9375;
% From 1500-series pickup, measured at Inland Truck Parts, in inches
l_fw_g=1.46875;
```

```
% From 1500-series pickup, measured at Inland Truck Parts, in inches
t_p = 12;
% From 1500-series pickup, measured at Inland Truck Parts
if g_r<G
   l_fw_p=l_fw_p*G/g_r;
   t_p = t_p*G/g_r;
elseif g_r>G
   l_fw_g=l_fw_g*g_r/G;
end
psi_p=acos(1.75/2.9375);
% From 1500-series pickup, measured at Inland Truck Parts.
psi_g=acos(1.3125/1.46875);
% From 1500-series pickup, measured at Inland Truck Parts.
L_fw = l_fw_p;
psi = psi_p;
R_f = 7.7782-0.1682*t_p/(2*r_p_p);
% linear least-squares fit from Table 12.5 in Dudley's Gear Handbook, 2nd Ed.
X=0.5;                                   % Assumed pinion not immersed.
torque_p = 15./0.746.*(X.*mu_oil+(1-
X).*mu_air)./mu_air.*(rpm./1000).^2./(1000./60.*2.*pi()).*(2.*r_p_p./100).^4.*(5.*L_f
w./100.*(1+R_f./sqrt(tan(psi)))+2.*r_p_p./100);
% power in hp for radii and lengths in inches
torque_p = torque_p * 745.7;
% converting hp*s into N-m from Moran and Shapiro's Fundamentals of
% Engineering Thermodynamics, 6th Ed., the list of conversion factors on the
% very first page.
eta_p_windage = 1 - torque_p./torque;
eta = eta .* eta_p_windage;
torque = torque .* eta_p_windage;
```

```
[eta,torque] = EtaNTorqCorrect(eta,torque,true);


% Next, the gear pair sliding friction

% f is the coefficient of friction.  A graph is available in Dudley's Gear
% Handbook for worm gears.  It is suggested that this graph is to be
% adapted to hypoid gears.  f is a function of four parameters that
% determine sliding speed.

hypoidspd=1;
dx_dt=rpm.*sqrt((r_o_p-r_p_p)^2+(l_fw_p-l_fw_g)^2/4)/12.*t_p*hypoidspd;      % In fpm.

% A curve fit for f as a function of sliding speed allows conversion of
% sliding speed to f.
f=0.8./(dx_dt+100).^0.5;  % Curve fit to Dudley's Gear Handbook, Figure 12.8.

% Calculation of efficiency, once all parameters are ascertained.
eta_gear = (cos(phi_n)+f.*tan(psi_g))./(cos(phi_n)+f.*tan(psi_p));
% Adapted from Equation 12.36 of Dudley's Gear Handbook, 2nd ed.

% Finally, all values need to be updated.
eta = eta.*eta_gear;
torque = torque.*g_r.*eta_gear;
rpm = rpm./g_r;
[eta,torque] = EtaNTorqCorrect(eta,torque,true);
```

```
% Finally, the windage on the downstream gear
% Windage calculation based on bevel gear windage calculation available
% from Dudley's Gear Handbook, 2nd ed.

psi=psi_g;
r_p_g = r_p_p * g_r;
r_i_g = 16.25/2/pi();
% From 1500-series pickup, measured at Inland Truck Parts
X=0.2;                          % Assumed 20% of crown gear immersed.
torque_g = 15./0.746.*(X.*mu_oil+(1-
X).*mu_air)./mu_air.*(rpm./1000).^2./(1000./60.*2.*pi()).*(2*r_p_g/100)^4.*(5*L_fw/1
00*(1+R_f/sqrt(tan(psi)))+2*(r_p_g^5-r_i_g^5)/r_p_g^4/100);
% power in hp for radii and lengths in inches
torque_g = torque_g * 745.7;
% converting hp*s into N-m from Moran and Shapiro's Fundamentals of
% Engineering Thermodynamics, 6th Ed., the list of conversion factors on the
% very first page.
eta_g_windage = 1 - torque_g./torque;
eta = eta .* eta_g_windage;
torque = torque .* eta_g_windage;
[eta,torque] = EtaNTorqCorrect(eta,torque,true);



% SPHERICAL WINDAGE



%   input radii of spheres, viscosity of lubricant.
X=0.1;
```

```
r=r_i_g*0.0254;          % in meters
R=r+0.0254;        % Assumed 1 inch clearance, in meters
if g_r>G
    r=r*g_r/G;
    R=R*g_r/G;
end
eta_sphere = 1 - (4.*pi().^2./45.*(X.*mu_oil+(1-X).*mu_air).*rpm.*r.^4./(R-r)) ./ torque;
eta = eta .* eta_sphere;
torque = torque .* eta_sphere ./2;      % Torque split
[eta,torque] = EtaNTorqCorrect(eta,torque,true);



% CYLINDRICAL WINDAGE



% Specifies angular velocity in rad/s from rpm in min^-1.
omega = 2.*pi()./60.*rpm;

% Gathers other required information
d = 1.25*0.0254;        % Measured from 1500-series at ITP.  In meters.
a = (2.5*.0254-d)/2;
% Computes clearance from previous inputs.  Measured from 1500-series @ ITP.
% In meters.
L = 3*0.0254;                    % Presumed 3 inches, in meters.

% Calculates windage efficiency from torque decrement
eta_windage2 = 1-(pi()./4.*mu.*D.^3.*L./a.*omega)./torque;
% Example Problem 8.2, page 320 from Fox, McDonald, and Pritchard's
```

```
% Introduction to Fluid Mechanics

% Adjusts values of interest
eta = eta.*eta_windage2;
torque = torque.*eta_windage2;
[eta,torque] = EtaNTorqCorrect(eta,torque,true);



% BEARING


lub_fric_fac_index=3;              % Defined for oil bath lubrication
nu = 36.6e-6;
% m^2/s, for gear oil at T = 70 degC, according to
% http://www.lloydminsterheavyoil.com/transportscience.htm
mu=0.0018;                               % These values of data
lub_fric_fac=[4.2,6,8,9];                   % were obtained from
f_1=20;                                  % Khonsari and Booser's
% Applied Tribology, 2nd ed.
f_2=25;                                  % Tables 15.1, 2, and 3.
f_L=lub_fric_fac(lub_fric_fac_index);          % Defined
R_p=r_p_g;
numBearings=2;                           % Defined
P = torque.*R_p./cos(phi_n)./numBearings.*2;
% Load on the bearings, derived loosely from first principles
d=d*1e3;                                 % Defined (mm)
D=D*1e3;                                 % Defined (mm)
d_m=0.5*(d+D);                           % Defined; used in M_L
```

```
M_P=5e-4*mu*P*d;
% Equation 15.1, page 462 from Khonsari and Booser's Applied Tribology, 2nd
% ed.

[a,b]=size(eta);
% This block is required for the possibility of an array of efficiencies.
M_L=zeros(a,b);                                  % Pre-allocation for speed
for c=1:1:a
    for e=1:1:b
        if(2*pi()/60*rpm(c,e)*nu>2000)
            M_L(c,e)=1e-10*f_L*(2*pi()/60*rpm(c,e)*nu)^(2/3)*d_m^3;
            % Equation 15.3, page 463 from Khonsari and Booser's Applied
            % Tribology, 2nd ed.
        else
            M_L(c,e)=1.6e-8*f_L*d_m^3;
            % Equation 15.4, page 463 from Khonsari and Booser's Applied
            % Tribology, 2nd ed.
        end
    end
end

M_S=1e-3*(((d+D)/f_1)^2+f_2);
% Equation 15.5, page 464 from Khonsari and Booser's Applied Tribology, 2nd
% ed.
M = M_P+M_L+M_S;
% Equation 15.2, page 463 from Khonsari and Booser's Applied Tribology, 2nd
% ed.
eta_bearing2 = 1-M./torque;              % Defined
eta = eta.*eta_bearing2;                 % Derived from first principles
```

```
torque = torque.*eta_bearing2;          % Derived from first principles
[eta,torque] = EtaNTorqCorrect(eta,torque,true);



% SEAL


% input seal parameters based on Dr. Matthews' model for seals in Appendix
% C
r_shaft = d/2e3;                    % In meters
F_tan_prime = 50+170*sealstr;          % In N/m

% calculate seal efficiency based on Dr. Matthews' model for seals in
% Appendix C
torque_seal = 2*pi()*r_shaft^2*F_tan_prime;   % Specified in N-m
eta_seal2 = 1 - torque_seal./torque;

% calculate new torque and eta.
torque = torque.*eta_seal2;
eta = eta.*eta_seal2;
[eta,torque] = EtaNTorqCorrect(eta,torque,true);



% PREPARE FINAL QUANTITIES


torque = torque./peak_torque./g_r.*2;
[eta,torque] = EtaNTorqCorrect(eta,torque,false);
```

end

Section B.3: Chain Drive Model

```
function [ eta,torque,speed ] = EffDiff4WD( torque,speed,diffG )
%EffDiff4WD Acts as transfer case and calls EffDiffRear.m
%   Written by James Vaughn for TxDOT project 0-5974 and his thesis.
%   Last modified 29 August 2011

torque=torque.*0.99;
% 99% efficiency of the transfer case, and assuming engaged in 2WD mode.
[eta,torque,speed]=EffDiffRearLD(torque,speed,diffG);
eta=eta.*0.99;
% Must do this afterwards to prevent needing to store two efficiencies.

end
```

Section B.4: Heavy-Duty Conventional Differential Model

```
function [ eta,torque,rpm ] = EffDiffRearHD( torque,rpm,g_r )
%EFFDIFFREARHD Efficiency of differential of rear axle 2WD vehicle
%
%   Written by James Vaughn as part of a thesis on behalf of the Texas
%   Department of Transportation, Professor Ronald Matthews, and the VCOST
%   and fuel economy models.  Last edited 31 July 2011.
%
%   Inputs:
%   torque - array of fractional torque from transmission output.
%            quotient of transmission output torque divided by peak
%            engine torque and highest possible transmission gear ratio.
%            (>0, please)
%   rpm - array of rotational speed of transmission output shaft, in rpm.
%         (>0, please.)
%   g_r - gear ratio (>=1, please.)
%
%   Outputs:
%   eta - array of efficiency of differential, fractional quantity.
%   torque - array of fractional torque at differential outputs.
%            quotient of differential output torque divided by peak
%            engine torque, highest possible transmission gear ratio,
%            and differential gear ratio.
%   rpm - array of rotational speed of differential output shafts, in rpm.
%
%   Note: requires access to walthersEqn.m.

% INITIALIZE PARAMETERS
```

```
[a,b]=size(torque);
% This block is required for the possibility of an array of efficiencies.
eta = ones(a,b);
%peak_torque = 4880;
% 3600 ft-lbf converted to N-m, based on MaxxForce 7 diesel engine
% ~ 600 ft-lbf * 6 (torque converter * first gear transmission gear ratio)
peak_torque = 32000;
% 2000 ft-lbf * 12 gear ratio, converted to N-m.
% Rough order-of-magnitude estimate from Dr. Dardalis.
torque = torque.*peak_torque;
[~,mu_air,~]=walthersEqn(0);
% Properties of air at operating temperature and other conditions;
% unnecessary parameters suppressed.
[~,mu_oil,~]=walthersEqn(3);
% Properties of 80W90 gear oil at operating temperature and other conditions;
% unnecessary parameters suppressed.
r_o_p=(1.9375+2.84375)/4*1.27576*41/12/3.9;
% From 1500-series pickup, measured at Inland Truck Parts, in inches;
% adjusted to images for heavy-duty differential
r_p_p=r_o_p-0.1875*1.27576*41/12/3.9;
% From 1500-series pickup, measured at Inland Truck Parts, in inches;
% adjusted to images for heavy-duty differential
phi_n=20*pi()/180;
% Assumed that the normal pressure angle is near 20 degrees
G=3.90;
% From heavy-duty differential, measured at Inland Truck Parts
if g_r<G
```

```
    r_o_p = r_o_p * G / g_r;
    r_p_p = r_p_p * G / g_r;
end



% SEAL



% input seal parameters based on Dr. Matthews' model for seals in Appendix
% C
r_shaft = 0.028575;
% Shaft radius, in meters (1.125 inch radius) for heavy-duty vehicles
F_tan_prime = 50+170*1;          % In N/m

% calculate seal efficiency based on Dr. Matthews' model for seals in
% Appendix C
torque_seal = 2*pi()*r_shaft^2*F_tan_prime;     % Specified in N-m
eta_seal = 1 - torque_seal./torque;

% calculate new torque and eta.
torque = torque.*eta_seal;
eta = eta.*eta_seal;
[eta,torque] = EtaNTorqCorrect(eta,torque,true);




% BEARING
```

```
lub_fric_fac_index=3;              % Defined for oil bath lubrication
nu = 36.6e-6;
% m^2/s, for gear oil at T = 70 degC, according to
% http://www.lloydminsterheavyoil.com/transportscience.htm
mu=0.0018;                               % These values of data
lub_fric_fac=[4.2,6,8,9];                    % were obtained from
f_1=20;                                  % Khonsari and Booser's
% Applied Tribology, 2nd ed.
f_2=25;                                  % Tables 15.1, 2, and 3.
f_L=lub_fric_fac(lub_fric_fac_index);          % Defined
R_p=r_p_p;                               % Defined
numBearings=1;                             % Defined
P = torque.*R_p./cos(phi_n)./numBearings;
% Load on the bearings, derived loosely from first principles
d=r_shaft*2e3;                             % Unit conversion (mm)
D=81/68*d;
% Assumption based on 2005 Chevy Equinox bearings (mm)
d_m=0.5*(d+D);                            % Defined; used in M_L
M_P=5e-4*mu*P*d;
% Equation 15.1, page 462 from Khonsari and Booser's Applied Tribology, 2nd
% ed.

M_L=zeros(a,b);
% Pre-allocation for speed; a and b were defined at the beginning.
for c=1:1:a
   for e=1:1:b
      if(2*pi()/60*rpm(c,e)*nu>2000)
         M_L(c,e)=1e-10*f_L*(2*pi()/60*rpm(c,e)*nu)^(2/3)*d_m^3;
         % Equation 15.3, page 463 from Khonsari and Booser's Applied
```

```
            % Tribology, 2nd ed.
        else
            M_L(c,e)=1.6e-8*f_L*d_m^3;
            % Equation 15.4, page 463 from Khonsari and Booser's Applied
            % Tribology, 2nd ed.
        end
    end
end

M_S=1e-3*(((d+D)/f_1)^2+f_2);
% Equation 15.5, page 464 from Khonsari and Booser's Applied Tribology, 2nd
% ed.
M = M_P+M_L+M_S;
% Equation 15.2, page 463 from Khonsari and Booser's Applied Tribology, 2nd
% ed.
eta_bearing = 1-M./torque;              % Defined
eta = eta.*eta_bearing;                 % Derived from first principles
torque = torque.*eta_bearing;           % Derived from first principles
[eta,torque] = EtaNTorqCorrect(eta,torque,true);



% CYLINDRICAL WINDAGE



% Specifies angular velocity in rad/s from rpm in min^-1.
omega = 2.*pi()./60.*rpm;

% Gathers other required information
```

```
% d recycled from above, converted to meters.
d = d/1e3;
D = D/1e3;
a = (D-d)/2;
% Computes clearance from previous inputs, recycling information from above.
L = 0.1016;
% 4 inches converted to meters, estimated from pictures taken at Inland Truck
% Parts.
mu = mu_air;                    % Can be air or oil lubricant

% Calculates windage efficiency from torque decrement
eta_windage = 1-(pi()./4.*mu.*d.^3.*L./a.*omega)./torque;
% Example Problem 8.2, page 320 from Fox, McDonald, and Pritchard's
% Introduction to Fluid Mechanics

% Adjusts values of interest
eta = eta.*eta_windage;
torque = torque.*eta_windage;
[eta,torque] = EtaNTorqCorrect(eta,torque,true);



% HYPOID GEAR PAIR



% First, the windage on the upstream pinion
% Windage calculation based on bevel gear windage calculation available
% from Dudley's Gear Handbook, 2nd ed.
```

```
% Psi's are helix angles of the pinion and gear and are estimated by
% geometric relationships.
l_fw_p=2.9375*1.27576*41/12/3.9;
% From 1500-series pickup, measured at Inland Truck Parts, in inches;
% adjusted to images for heavy-duty differential
l_fw_g=1.46875*1.27576;
% From 1500-series pickup, measured at Inland Truck Parts, in inches;
% adjusted to images for heavy-duty differential
t_p = 12;
% From heavy-duty differential, measured at Inland Truck Parts
if g_r<G
   l_fw_p=l_fw_p*G/g_r;
   t_p = t_p*G/g_r;
elseif g_r>G
   l_fw_g=l_fw_g*g_r/G;
end
psi_p=acos(1.75/2.9375);
% From 1500-series pickup, measured at Inland Truck Parts.
% Assumed to be close.
psi_g=acos(1.3125/1.46875);
% From 1500-series pickup, measured at Inland Truck Parts.
% Assumed to be close.
L_fw = l_fw_p;
psi = psi_p;
R_f = 7.7782-0.1682*t_p/(2*r_p_p);
% linear least-squares fit from Table 12.5 in Dudley's Gear Handbook, 2nd Ed. X=0;
% Assumed pinion not immersed.
```

```
torque_p = 15./0.746.*(X.*mu_oil+(1-
X).*mu_air)./mu_air.*(rpm./1000).^2./(1000.*2.*pi()./60).*(2.*r_p_p./100).^4.*(5.*L_f
w./100.*(1+R_f./sqrt(tan(psi)))+2.*r_p_p./100);
% power in hp for radii and lengths in inches
torque_p = torque_p * 745.7;
% converting hp*s into N-m from Moran and Shapiro's Fundamentals of
% Engineering Thermodynamics, 6th Ed., the list of conversion factors on the
% very first page.
eta_p_windage = 1 - torque_p./torque;
eta = eta .* eta_p_windage;
torque = torque .* eta_p_windage;
[eta,torque] = EtaNTorqCorrect(eta,torque,true);


% Next, the gear pair sliding friction

% f is the coefficient of friction.  A graph is available in Dudley's Gear
% Handbook for worm gears.  It is suggested that this graph is to be
% adapted to hypoid gears.  f is a function of four parameters that
% determine sliding speed.

dx_dt=rpm./12.*sqrt((r_o_p-r_p_p)^2+(l_fw_p-l_fw_g)^2/4).*t_p;      % In fpm.

% A curve fit for f as a function of sliding speed allows conversion of
% sliding speed to f.
f=0.8./(dx_dt+100).^0.5;
% Curve fit to Dudley's Gear Handbook, Figure 12.8.

% Calculation of efficiency, once all parameters are ascertained.
```

```
eta_gear = (cos(phi_n)+f.*tan(psi_g))./(cos(phi_n)+f.*tan(psi_p));
% Adapted from Equation 12.36 of Dudley's Gear Handbook, 2nd ed.

% Finally, all values need to be updated.
eta = eta.*eta_gear;
torque = torque.*g_r.*eta_gear;
rpm = rpm./g_r;
[eta,torque] = EtaNTorqCorrect(eta,torque,true);


% Finally, the windage on the downstream gear
% Windage calculation based on bevel gear windage calculation available
% from Dudley's Gear Handbook, 2nd ed.

psi=psi_g;
r_p_g = r_p_p * g_r;
r_i_g = 16.25/2/pi()*1.27576;
% From 1500-series pickup, measured at Inland Truck Parts;
% adjusted to images for heavy-duty differential
X=0.03;                         % Assumed 3% of crown gear immersed.
torque_g = 15./0.746.*(X.*mu_oil+(1-
X).*mu_air)./mu_air.*(rpm./1000).^2./(1000./60.*2.*pi()).*(2*r_p_g/100)^4.*(5*L_fw/1
00*(1+R_f/sqrt(tan(psi)))+2*(r_p_g^5-r_i_g^5)/r_p_g^4/100);
% power in hp for radii and lengths in inches
torque_g = torque_g * 745.7;
% converting hp into Watts (N-m/s) from Moran and Shapiro's Fundamentals of
% Engineering Thermodynamics, 6th Ed., the list of conversion factors on the
% very first page.
eta_g_windage = 1 - torque_g./torque;
```

```
eta = eta .* eta_g_windage;
torque = torque .* eta_g_windage;
[eta,torque] = EtaNTorqCorrect(eta,torque,true);



% SPHERICAL WINDAGE


%   input radii of spheres, viscosity of lubricant.
X=0;
r=r_i_g*0.0254;          % in meters
R=r+0.0254;        % Assumed 1 inch clearance, in meters
eta_sphere = 1 - (4.*pi().^2./45.*(X.*mu_oil+(1-X).*mu_air).*rpm.*r.^4./(R-r)) ./ torque;
eta = eta .* eta_sphere;
torque = torque .* eta_sphere ./2;     % Torque split
[eta,torque] = EtaNTorqCorrect(eta,torque,true);



% CYLINDRICAL WINDAGE


% Specifies angular velocity in rad/s from rpm in min^-1.
omega = 2.*pi()./60.*rpm;

% Gathers other required information
d = 1.25*1.27576*0.0254;
% Measured from 1500-series at ITP, modified for heavy-duty.  In meters.
```

```
a = (2.5*1.27576*.0254-d)/2;
% Computes clearance from previous inputs.  Inferred from 1500-series @ ITP.
% In meters.
L = 3*0.0254;                    % Presumed 3 inches, in meters.

% Calculates windage efficiency from torque decrement
eta_windage = 1-(pi()./4.*mu.*D.^3.*L./a.*omega)./torque;
% Example Problem 8.2, page 320 from Fox, McDonald, and Pritchard's
% Introduction to Fluid Mechanics

% Adjusts values of interest
eta = eta.*eta_windage;
torque = torque.*eta_windage;
[eta,torque] = EtaNTorqCorrect(eta,torque,true);



% BEARING


lub_fric_fac_index=3;              % Defined for oil bath lubrication
mu=0.0018;                               % These values of data
lub_fric_fac=[4.2,6,8,9];                    % were obtained from
f_1=20;                                 % Khonsari and Booser's
% Applied Tribology, 2nd ed.
f_2=25;                                 % Tables 15.1, 2, and 3.
f_L=lub_fric_fac(lub_fric_fac_index);          % Defined
R_p=r_p_g;                               % To be measured
numBearings=2;                             % Defined
```

```
P = torque.*R_p./cos(phi_n)./numBearings.*2;
% Load on the bearings, derived loosely from first principles
d=d*1e3;                              % Defined (mm)
D=D*1e3;                              % Defined (mm)
d_m=0.5*(d+D);                        % Defined; used in M_L
M_P=5e-4*mu*P*d;
% Equation 15.1, page 462 from Khonsari and Booser's Applied Tribology, 2nd
% ed.

[a,b]=size(eta);
% This block is required for the possibility of an array of efficiencies.
M_L=zeros(a,b);                       % Pre-allocation for speed
for c=1:1:a
    for e=1:1:b
        if(2*pi()/60*rpm(c,e)*nu>2000)
            M_L(c,e)=1e-10*f_L*(2*pi()/60*rpm(c,e)*nu)^(2/3)*d_m^3;
            % Equation 15.3, page 463 from Khonsari and Booser's Applied
            % Tribology, 2nd ed.
        else
            M_L(c,e)=1.6e-8*f_L*d_m^3;
            % Equation 15.4, page 463 from Khonsari and Booser's Applied
            % Tribology, 2nd ed.
        end
    end
end

M_S=1e-3*(((d+D)/f_1)^2+f_2);
% Equation 15.5, page 464 from Khonsari and Booser's Applied Tribology, 2nd
% ed.
```

```
M = M_P+M_L+M_S;
% Equation 15.2, page 463 from Khonsari and Booser's Applied Tribology, 2nd
% ed.
eta_bearing = 1-M./torque;            % Defined
eta = eta.*eta_bearing;               % Derived from first principles
torque = torque.*eta_bearing;         % Derived from first principles
[eta,torque] = EtaNTorqCorrect(eta,torque,true);


% SEAL

% input seal parameters based on Dr. Matthews' model for seals in Appendix
% C
r_shaft = d/2e3;                  % In meters
F_tan_prime = 50+170*1;           % In N/m

% calculate seal efficiency based on Dr. Matthews' model for seals in
% Appendix C
torque_seal = 2*pi()*r_shaft^2*F_tan_prime;    % Specified in N-m
eta_seal = 1 - torque_seal./torque;

% calculate new torque and eta.
torque = torque.*eta_seal;
eta = eta.*eta_seal;
[eta,torque] = EtaNTorqCorrect(eta,torque,true);


% PREPARE FINAL QUANTITIES
```

```
torque = torque./peak_torque./g_r*2;
[eta,torque] = EtaNTorqCorrect(eta,torque,false);


end
```

Section B.5: Two-Speed Gearbox Model

```
function [ trans_efficiency_aux,torque_d_OS_aux,RPM_O_aux ] = Auxiliary_modified(
input_torque_aux,input_RPM_aux,toospd_lo_gear,gear,diffG )
%AUXILIARY_MODIFIED 2-speed gearbox simulation for dual differential
%   Kyung Jin Kim's program was adapted by James Vaughn to suit the
%   purposes of modeling a dual differential like those found in some
%   heavy-duty vehicles.  Note that Kyung Jin Kim wrote the original program,
%   and in some cases, it is easy to see where James Vaughn edited the code.
%   Last edited by James Vaughn on 29 August 2011
%   Inputs:
%   input_torque_aux - torque fraction of the full capacity of the 2-speed
%                      gearbox.  Can be an array.
%   input_RPM_aux - transmission output rotational speed, in rpm.  Can be
%                   an array.
%   G_i_aux - first gear ratio of the two reductions that occur when the
%             low gear is engaged.  43/34 for measured gearbox.
%   G_L_aux - second gear ratio of the two reductions that occur when the
%             low gear is engaged.  62/16 for measured gearbox.  Suggest
%             maintaining ratio of gear ratios if only total gear ratio is
%             known.
%   gear - 1 if low gear, 2 if high gear
%   diffG - gear ratio of the rear-axle differential.
%
%   Outputs:
%   trans_efficiency_aux - fractional efficiency of the 2-speed gearbox.
%                          Can be an array.
%   torque_d_OS_aux - torque fraction after the 2-speed gearbox.  Can be an
%                     array.
```

```
%   RPM_O_aux - gearbox output rotational speed, in rpm.  Can be an array.

%%%% This program was coded by Kyung Jin Kim, for TxDOT VCost model %%%%
%%%% This program was modeled Eaton Fuller FRO16210C 10-speed manual
transmission %%%%
%%%% All constants were acquired from the manufacturer. %%%
%%%% The constants that were not able to acquired from the manufactuer were
assumed %%%

%%%% This program described the auxiliary stage of the transmission, which
%%%% is the second block of the transmission



%clc
%clear

% JRV Constants
peak_torque=5794.088;
% Peak torque from equivalent transmission to E.F. FRO16210C.
% 1650 ft-lbf * 2.59 (first gear ratio of the manual 5-speed transmission)
% converted to N-m
input_torque_aux=input_torque_aux.*peak_torque;

% General Constants
alpha_n = 20;                   % Normal Pressure Angle - Assumed
C = 149.5044;               % Center Distance, 5.886 inches, measured data
mu_fGP = 0.06;                  % Coefficient of friction - Assumed
X = 0.03;
% Portion of gear area submerged under the oil level that exposed to the air
```

```
C_E = 0.7;                          % Constants for loose - Assumed


% Seal Constants
F_tan = 150;
% Tangential friction circumference (median, N/m) - Assumed as nitrile
% material seal, value of oil pressure at 50 psi

D_sh_output = 70.0532;
% output shaft seal outer diameter (mm), shaft diameter = 2.758 inches
r_sh_output = D_sh_output/2;        % output shaft seal radius (mm)


%%%%% Seal losses
torqueloss_seal_output = 2* pi * (r_sh_output * (10^-3))^2 * F_tan;
% Output shaft seal loss, (only output seal exist)


% Gear Constants
t_D_i_aux = 34;
% Initial drive gear teeth number, Input shaft (Auxiliary)
t_d_i_aux = 43;
% Initial driven gear teeth number, Counter shaft (Auxiliary)
G_i_aux = t_d_i_aux/t_D_i_aux;
% Initial gear ratio (Auxiliary)
gamma_i_aux = 32.5;
% Initial drive/driven gear pair helix angle (degree) - measured
alpha_t_i_aux = atand((tand(alpha_n)/cosd(gamma_i_aux)));
% Initial drive/driven gear transverse pressure angle
```

```
R_p_D_i_aux = C/(1+(t_d_i_aux/t_D_i_aux));                              % Initial
drive gear pitch radius (mm) (Auxiliary)
R_b_D_i_aux = R_p_D_i_aux * cosd(alpha_n);
% Initial drive gear base radius (mm) (Auxiliary)
R_p_d_i_aux = C - R_p_D_i_aux;
% Initial driven gear pitch radius (mm) (Auxiliary)
R_b_d_i_aux = R_p_d_i_aux * cosd(alpha_n);
% Initial driven gear base radius (mm) (Auxiliary)
OD_D_i_aux = 142.24;
% Outer Diameter of initial drive gear (mm), 4.374 inches (Auxiliary)
OD_d_i_aux = 175.1584;
% Outer Diameter of initial driven gear (mm), 7.73 inches (Auxiliary)
X_D_i_aux = 0.14286904;
% Portion(X) of initial drive gear area submerged under the oil level (Auxiliary)
X_d_i_aux_lower = 0.492754481;
% Portion(X) of initial driven gear area submerged under the oil level, lower
% counter shaft (Auxiliary)
L_FW_i_aux = 33.3756;
% Face width of initial drive gear (mm), 1.203 inches (Auxiliary)

% JRV: Old assumption for: Accounting for different initial gear ratio
% if G_i_aux>(t_d_i_aux/t_D_i_aux)
% Driven gear will grow
%    R_p_D_i_aux=R_p_D_i_aux*G_i_aux/(t_d_i_aux/t_D_i_aux);
%    R_b_D_i_aux=R_b_D_i_aux*G_i_aux/(t_d_i_aux/t_D_i_aux);
%    OD_D_i_aux=OD_D_i_aux*G_i_aux/(t_d_i_aux/t_D_i_aux);
% else
% Drive gear will grow
%    R_p_d_i_aux=R_p_d_i_aux/G_i_aux*(t_d_i_aux/t_D_i_aux);
```

```
%     R_b_d_i_aux=R_b_d_i_aux/G_i_aux*(t_d_i_aux/t_D_i_aux);
%     OD_d_i_aux=OD_d_i_aux/G_i_aux*(t_d_i_aux/t_D_i_aux);
% end

t_D_L_aux = 16;
% Low drive gear teeth number, Counter (Auxiliary)
t_d_L_aux = 62;
% Low driven gear teeth number, Output (Freewheeling)
G_L_aux=toospd_lo_gear/G_i_aux;
gamma_L_aux = 12.5;
% Low drive/driven gear helix angle (degree) - measured
alpha_t_L_aux = atand((tand(alpha_n)/cosd(gamma_L_aux)));
% 1st drive/driven gear transverse pressure angle
R_p_D_L_aux = C/(1+(t_d_L_aux/t_D_L_aux));                        % Low
drive gear pitch radius (mm)
R_b_D_L_aux = R_p_D_L_aux * cosd(alpha_n);
% Low drive gear base radius (mm)
R_p_d_L_aux = C - R_p_D_L_aux;
% Low driven gear pitch radius (mm)
R_b_d_L_aux = R_p_d_L_aux * cosd(alpha_n);
% Low driven gear base radius (mm)
OD_D_L_aux = 69.469;
% Outer diameter of 1st drive gear (mm)
OD_d_L_aux = 239.776;
% Outer diameter of 1st driven gear (mm), 9.585 inches
X_D_L_aux_lower = 0.48551962;
% Portion(X) of low drive gear area submerged under the oil level (Auxiliary)
X_d_L_aux = 0.279481458;
% Portion(X) of Low driven gear area submerged under the oil level
```

```
% (Auxiliary)
L_FW_L_aux = 59.8424;
% Face width of 1st drive gear (mm), used driven gear facewidth data, 1.427
% inches

% JRV: Accounting for different low gear ratio by changing driven gear size
R_p_D_L_aux=R_p_D_L_aux*G_L_aux/(t_d_L_aux/t_D_L_aux);
R_b_D_L_aux=R_b_D_L_aux*G_L_aux/(t_d_L_aux/t_D_L_aux);
OD_D_L_aux=OD_D_L_aux*G_L_aux/(t_d_L_aux/t_D_L_aux);

%note: all gear on upper counter shaft (initial driven gear, low drive
%gear) is not submerged under the oil level, therefore X=0.03 is applied.


beta_i_D_aux = (sqrt(((OD_D_i_aux)^2) - ((2*R_b_D_i_aux)^2)) - (2 * R_p_D_i_aux *
sind(alpha_t_i_aux)))/(2*R_b_D_i_aux);
beta_i_d_aux = (sqrt(((OD_d_i_aux)^2) - ((2*R_b_d_i_aux)^2)) - (2 * R_p_d_i_aux *
sind(alpha_t_i_aux)))/(2*R_b_D_i_aux);
GPEff_i_aux = 1- ((cosd(alpha_t_i_aux)/(cosd(alpha_n)*cosd(gamma_i_aux))) * (1 +
(1/G_i_aux)) * (mu_fGP/2) * (((beta_i_D_aux^2) + (beta_i_d_aux^2))/(beta_i_D_aux +
beta_i_d_aux)));
% Initial gear pair efficiency

beta_L_D_aux = (sqrt(((OD_D_L_aux)^2) - ((2*R_b_D_L_aux)^2)) - (2 *
R_p_D_L_aux * sind(alpha_t_L_aux)))/(2*R_b_D_L_aux);
beta_L_d_aux = (sqrt(((OD_d_L_aux)^2) - ((2*R_b_d_L_aux)^2)) - (2 * R_p_d_L_aux
* sind(alpha_t_L_aux)))/(2*R_b_D_L_aux);
```

```
GPEff_L_aux = 1- ((cosd(alpha_t_L_aux)/(cosd(alpha_n)*cosd(gamma_L_aux))) * (1 +
(1/G_L_aux)) * (mu_fGP/2) * (((beta_L_D_aux^2) + (beta_L_d_aux^2))/(beta_L_D_aux
+ beta_L_d_aux)));
% 1st gear pair efficiency


%%% Input variables
%T_ambient = input('Ambient Air Temperature (F)?');                % Ambient
temperature in Fahrenheit
T_ambient = 72;
%T_oil = (((T_ambient+50)-32)*(5/9)) + 273.1;                   % Oil Temperature
in Transmission
T_oil = (T_ambient-32)*5/9+273.1+50;
T_air = T_oil;
% Air Temperature in Transmission
rho_air = 101325 / (287.058 * T_air);
% Assumed as ideal gas. Dry air condition with 1 atm. Unit is kg/m3.
mu_air = (-1.1555*(10^-14)*T_air^3) + (9.5728 * (10^-11) * T_air^2) + (3.7604 * (10^-
8) * T_air) - (3.4484 * (10^-6));
% Kinematic viscosity of Air vs. Temperature. Unit is m2/sec.
mu_air_dynamic = (mu_air * rho_air) * (1000);
% Dynamic viscosity of aire on Temperature (K), the unit is cP (centipose)

rho_oil = ((-0.0007*(T_oil-273.1))+0.860)* 999;
% Density of Oil (using specific gravity)on Temperature (C), Castrol Syngear
% CD-50 (satisfied the Eaton PS-164 rev 7 specs), unit is kg/m^3
A_cons = (T_oil/373.1)^(-2.965198089);
% Calculation result from Walther's equation
mu_oil = ((18.3^A_cons) -0.7) * (10^-6);
```

```
% Kinematic viscosity of oil on Temperature (K), the unit is m^2/s (converted
% from cST)
mu_oil_dynamic = (mu_oil * rho_oil) * (1000);
% Dynamic viscosity of oil on Temperature (K), the unit is cP (centipose)

%input_torque_aux = input('Output torque of main stage in Nm?');            %
Transmission max torque is 2167 Nm (1600 lbft)
%input_RPM_aux = input('Output rpm of main stage?');                      % Engine red
zone starts from 2200 rpm (assumed)

RPM_aux = input_RPM_aux;
% Aux. stage input shaft RPM
RPM_C_aux = RPM_aux./G_i_aux;                                          % Aux. stage counter
shaft RPM

%%% Above code should be erased when I combine with the main stage block.



%%% Windage losses calculation
% JRV edit.  Lots of dots for matrix multiplication.
torque_oil_i_D_aux = 1.06948 .* 10.^-4 .* rho_oil .* C_E .* (1+ (2.3 .*
(L_FW_i_aux./R_p_D_i_aux))) .* (RPM_aux.^1.85) .* (OD_D_i_aux .* 10.^-3).^4.7 .*
(mu_oil).^0.15;
% Windage torque loss from oil of initial drive gear
torque_air_i_D_aux = 1.06948 .* 10.^-4 .* rho_air .* C_E .* (1+ (2.3 .*
(L_FW_i_aux./R_p_D_i_aux))) .* (RPM_aux.^1.85) .* (OD_D_i_aux .* 10.^-3).^4.7 .*
(mu_air).^0.15;
% Windage torque loss from air of initial drive gear
```

```
torque_i_D_aux = (X_D_i_aux .*torque_oil_i_D_aux) + ((1-
X_D_i_aux).*torque_air_i_D_aux);
% Windage torque loss on initial drive gear

torque_oil_i_d_aux_upper = 1.06948 .* 10.^-4 .* rho_oil .* C_E .* (1+ (2.3 .*
(L_FW_i_aux./R_p_d_i_aux))) .* ((RPM_C_aux).^1.85) .* (OD_d_i_aux .* 10.^-3).^4.7
.* (mu_oil).^0.15;
% Windage torque loss from oil of initial driven gear
torque_air_i_d_aux_upper = 1.06948 .* 10.^-4 .* rho_air .* C_E .* (1+ (2.3 .*
(L_FW_i_aux./R_p_d_i_aux))) .* ((RPM_C_aux).^1.85) .* (OD_d_i_aux .* 10.^-3).^4.7
.* (mu_air).^0.15;
% Windage torque loss from air of initial driven gear
torque_i_d_aux_upper = (X .* torque_oil_i_d_aux_upper) + ((1-
X).*torque_air_i_d_aux_upper);                                              %
Windage torque loss on initial driven gear

torque_oil_i_d_aux_lower = 1.06948 .* 10.^-4 .* rho_oil .* C_E .* (1+ (2.3 .*
(L_FW_i_aux./R_p_d_i_aux))) .* ((RPM_C_aux).^1.85) .* (OD_d_i_aux .* 10.^-3).^4.7
.* (mu_oil).^0.15;
% Windage torque loss from oil of initial driven gear
torque_air_i_d_aux_lower = 1.06948 .* 10.^-4 .* rho_air .* C_E .* (1+ (2.3 .*
(L_FW_i_aux./R_p_d_i_aux))) .* ((RPM_C_aux).^1.85) .* (OD_d_i_aux .* 10.^-3).^4.7
.* (mu_air).^0.15;
% Windage torque loss from air of initial driven gear
torque_i_d_aux_lower = (X_d_i_aux_lower .* torque_oil_i_d_aux_lower) + ((1-
X_d_i_aux_lower).*torque_air_i_d_aux_lower);                                %
Windage torque loss on initial driven gear
```

```
torque_oil_L_D_aux_upper = 1.06948 .* 10.^-4 .* rho_oil .* C_E .* (1+ (2.3 .*
(L_FW_L_aux./R_p_D_L_aux))) .* ((RPM_C_aux).^1.85) .* (OD_D_L_aux .* 10.^-
3).^4.7 .* (mu_oil).^0.15;
% Windage torque loss from oil of 1st drive gear
torque_air_L_D_aux_upper = 1.06948 .* 10.^-4 .* rho_air .* C_E .* (1+ (2.3 .*
(L_FW_L_aux./R_p_D_L_aux))) .* ((RPM_C_aux).^1.85) .* (OD_D_L_aux .* 10.^-
3).^4.7 .* (mu_air).^0.15;
% Windage torque loss from air of 1st drive gear
torque_L_D_aux_upper = (X .* torque_oil_L_D_aux_upper) + ((1-
X).*torque_air_L_D_aux_upper);                                              %
Windage torque loss on 1st drive gear

torque_oil_L_D_aux_lower = 1.06948 .* 10.^-4 .* rho_oil .* C_E .* (1+ (2.3 .*
(L_FW_L_aux./R_p_D_L_aux))) .* ((RPM_C_aux).^1.85) .* (OD_D_L_aux .* 10.^-
3).^4.7 .* (mu_oil).^0.15;
% Windage torque loss from oil of 1st drive gear
torque_air_L_D_aux_lower = 1.06948 .* 10.^-4 .* rho_air .* C_E .* (1+ (2.3 .*
(L_FW_L_aux./R_p_D_L_aux))) .* ((RPM_C_aux).^1.85) .* (OD_D_L_aux .* 10.^-
3).^4.7 .* (mu_air).^0.15;
% Windage torque loss from air of 1st drive gear
torque_L_D_aux_lower = (X_D_L_aux_lower .* torque_oil_L_D_aux_lower) + ((1-
X_D_L_aux_lower).*torque_air_L_D_aux_lower);                                %
Windage torque loss on 1st drive gear


%%%%%%%%%%%%% Torque of Inital drive gear
torque_D_IS_aux = input_torque_aux - torque_i_D_aux;
% Torque delivered to initial drive gear on input shaft, No input seal for
% Aux. stage, No input support bearing existed in the real FRO16210C
```

```
% transmission



%%% Counter shaft Bearing losses properties
R_C_aux = 1;
% Counter shafts rotates counter clockwise
H_C_aux = 1;
% "Hands" of Counter shaft, right hand helix

% Tapered Roller Bearings #7, #8, #9, #10 on auxiliary lower and upper
% counter shaft(front & rear) properties
K78910 = 1.49;
% factor value of bearing #7, #8, #9, #10
G78910 = 26.4;
% geometry factor value of bearing #7, #8, #9, #10

% Double Tapered Roller Bearing (Double Outer Race Series) #11 properties
K11 = 1.52;
% factor value of bearing #11
G11 = 77.2;
% geometry factor value of bearing #11

R_IO_aux = 1;
H_IO_aux = 1;



%%% Distances between bearing and gear, bearing to bearing (Measured)
```

```
L_B7_d = 28.6512;
% Distance from Bearing #7 to input driven gear on aux. stage lower counter
% shaft (mm), 1.128 inches
L_B7_LD = 128.8288;
% Distance from Bearing #7 to low drive gear on aux. stage lower counter
% shaft (mm), 5.072 inches
L_B7_B8 = 165.10;
% Distance from Bearing #7 to Bearing #8 on aux. stage lower counter shaft
% (mm), 6.50 inches

L_d_B8 = L_B7_B8 - L_B7_d;
% Distance from input driven gear on aux. stage lower counter shaft to
% Bearing #8 (mm), 5.372 inches
L_LD_B8 = L_B7_B8 - L_B7_LD;
% Distance from low drive gear on aux. stage lower counter shaft to Bearing
% #8 (mm), 1.428 inches

L_B9_d = L_B7_d;
% Distance from Bearing #9 to input driven gear on aux. stage upper counter
% shaft (mm), 1.128 inches
L_B9_LD = L_B7_LD;
% Distance from Bearing #9 to low drive gear on aux. stage upper counter
% shaft (mm), 5.072 inches
L_B9_B10 = L_B7_B8;
% Distance from Bearing #9 to Bearing #10 on aux. stage upper counter shaft
% (mm), 6.50 inches

L_d_B10 = L_d_B8;
% Distance from input driven gear on aux. stage upper counter shaft to
```

```
% Bearing #10 (mm), 5.372 inches
L_LD_B10 = L_LD_B8;
% Distance from low drive gear on aux. stage lower counter shaft to Bearing
% #10 (mm), 1.428 inches

L_B11_aux = 103.80218;
% Distance from enf of the shaft to Bearing #11 on aux. stage output shaft
% (mm), 4.0867 inches
L_d_aux = 64.4398;
% Distance from end of the shaft to low speed driven gear on aux. stage
% output shaft (mm), 2.537 inches


%%%%% Input gear speed
%gear = input('Enter a gear speed 1 or 2 (put 1 for low gear, put 2 for high
%gear): ');
% Gear speed input (2 speed), need to select from one (for low gear) or two
% (for high gear).

switch gear
   case 1

      %%%%%%%%%%%%%% Torque delivered to the input driven gear on the
% counter shaft
      torque_d_IS_aux_lower = 0.5 .* G_i_aux .* torque_D_IS_aux .* GPEff_i_aux;
% Torque delivered to the initial driven gear on aux. stage lower counter
% shaft
      torque_d_IS_aux_upper = 0.5 .* G_i_aux .* torque_D_IS_aux .* GPEff_i_aux;
% Torque delivered to the initial driven gear on aux. stage upper counter
```

```
% shaft

        RPM_O_aux = RPM_C_aux./G_L_aux;
% Output shaft RPM (is counter shaft RPM divided by 1st gear pair ratio

        torque_oil_L_d_aux = 1.06948 .* 10.^-4 .* rho_oil .* C_E .* (1+ (2.3 .*
(L_FW_L_aux./R_p_d_L_aux))) .* ((RPM_C_aux./G_L_aux).^1.85) .* (OD_d_L_aux .*
10.^-3).^4.7 .* (mu_oil).^0.15;
% Windage torque loss from oil of 1st driven gear
        torque_air_L_d_aux = 1.06948 .* 10.^-4 .* rho_air .* C_E .* (1+ (2.3 .*
(L_FW_L_aux./R_p_d_L_aux))) .* ((RPM_C_aux./G_L_aux).^1.85) .* (OD_d_L_aux .*
10.^-3).^4.7 .* (mu_air).^0.15;
% Windage torque loss from air of 1st driven gear
        torque_L_d_aux = (X_d_L_aux .*torque_oil_L_d_aux) + ((1-
X_d_L_aux).*torque_air_L_d_aux);                                                %
Windage torque loss on 1st driven gear


        F_t_i_d_aux_lower =  (1000 .* torque_d_IS_aux_lower .*
R_C_aux)./(R_p_d_i_aux);
% Absolute tangential force of initial drive gear
        F_s_i_d_aux_lower =  (abs(F_t_i_d_aux_lower).*
tand(alpha_n)./cosd(gamma_i_aux));
% Absolute seperating force of initial drive gear
        F_a_i_d_aux_lower =  H_C_aux .* R_C_aux .* abs(F_t_i_d_aux_lower) .*
tand(gamma_i_aux);
% Absolute axial force of initial drive gear
```

```
    torque_D_L_aux_lower = torque_d_IS_aux_lower - torque_i_d_aux_lower -
torque_L_D_aux_lower;
% Assumed torque of 1st drive gear on counter shaft
    F_t_L_D_aux_lower =  (1000 .* torque_D_L_aux_lower .*
R_C_aux)./(R_p_D_L_aux);
% Absolute tangential force of 1st drive gear
    F_s_L_D_aux_lower =  (abs(F_t_L_D_aux_lower).*
tand(alpha_n)./cosd(gamma_L_aux));
% Absolute seperating force of 1st drive gear
    F_a_L_D_aux_lower =  H_C_aux .* R_C_aux .* abs(F_t_L_D_aux_lower) .*
tand(gamma_L_aux);
% Absolute axial force of 1st drive gear


    F_t_i_d_aux_upper =  (1000 .* torque_d_IS_aux_upper .*
R_C_aux)./(R_p_d_i_aux);
% Absolute tangential force of initial drive gear
    F_s_i_d_aux_upper =  (abs(F_t_i_d_aux_upper).*
tand(alpha_n)./cosd(gamma_i_aux));
% Absolute seperating force of initial drive gear
    F_a_i_d_aux_upper =  H_C_aux .* R_C_aux .* abs(F_t_i_d_aux_upper) .*
tand(gamma_i_aux);
% Absolute axial force of initial drive gear

    torque_D_L_aux_upper = torque_d_IS_aux_upper - torque_i_d_aux_upper -
torque_L_D_aux_upper;
% Assumed torque of 1st drive gear on counter shaft
    F_t_L_D_aux_upper =  (1000 .* torque_D_L_aux_upper .*
R_C_aux)./(R_p_D_L_aux);
```

```
% Absolute tangential force of 1st drive gear
        F_s_L_D_aux_upper =  (abs(F_t_L_D_aux_upper).*
tand(alpha_n)./cosd(gamma_L_aux));
% Absolute seperating force of 1st drive gear
        F_a_L_D_aux_upper =  H_C_aux .* R_C_aux .* abs(F_t_L_D_aux_upper) .*
tand(gamma_L_aux);
% Absolute axial force of 1st drive gear




        %%% Force momentume equations for tapered roller bearing #7 and #8
        F_apc_B7 = abs((-1/L_B7_B8).*((F_s_i_d_aux_lower .* L_d_B8) +
(F_s_L_D_aux_lower .* L_LD_B8) + (F_a_i_d_aux_lower .* R_p_d_i_aux) +
(F_a_L_D_aux_lower .* R_p_D_L_aux))) ;
% Force that is aligned with the plane of bearing #2(axial force), absolute
% value
        F_apc_B8 = abs((1/L_B7_B8) .* ((F_a_i_d_aux_lower .* R_p_d_i_aux) +
(F_a_L_D_aux_lower .* R_p_D_L_aux) - (F_s_i_d_aux_lower .* L_B7_d) -
(F_s_L_D_aux_lower .* L_B7_LD))) ;
% Force that is aligned with the plane of bearing #3(axial force), absolute
% value
        F_ppc_B7 = abs((1/L_B7_B8) .* ((F_t_i_d_aux_lower .* L_d_B8) +
(F_t_L_D_aux_lower .* L_LD_B8)));
% Force that is perpendicular to the plane of bearing #2(tangential force),
% absolute value
        F_ppc_B8 = abs((1/L_B7_B8) .* ((F_t_i_d_aux_lower .* L_B7_d) +
(F_t_L_D_aux_lower .* L_B7_LD)));
% Force that is perpendicular to the plane of bearing #3(tangential force),
% absolute value
```

```
    F_r_B7 = sqrt((F_apc_B7).^2 + (F_ppc_B7).^2);
% Radial force applying to bearing #2 (tapered roller bearing)
    F_r_B8 = sqrt((F_apc_B8).^2 + (F_ppc_B8).^2);
% Radial force applying to bearing #3 (tapered roller bearing)



    %%% Force momentume equations for tapered roller bearing #9 and #10
    F_apc_B9 = abs((1./L_B9_B10).*(-(F_s_i_d_aux_upper .* L_d_B10) -
(F_s_L_D_aux_upper .* L_LD_B10) - (F_a_i_d_aux_upper .* R_p_d_i_aux) -
(F_a_L_D_aux_upper .* R_p_D_L_aux))) ;
% Force that is aligned with the plane of bearing #2(axial force), absolute
% value
    F_apc_B10 = abs((1./L_B9_B10) .* ((F_a_i_d_aux_upper .* R_p_d_i_aux) +
(F_a_L_D_aux_upper .* R_p_D_L_aux) - (F_s_i_d_aux_upper .* L_B9_d) -
(F_s_L_D_aux_upper .* L_B9_LD))) ;
% Force that is aligned with the plane of bearing #3(axial force), absolute
% value
    F_ppc_B9 = abs((1./L_B9_B10) .* ((F_t_i_d_aux_upper .* L_d_B10) +
(F_t_L_D_aux_upper .* L_LD_B10)));
% Force that is perpendicular to the plane of bearing #2(tangential force),
% absolute value
    F_ppc_B10 = abs((1./L_B9_B10) .* ((F_t_i_d_aux_upper .* L_B9_d) +
(F_t_L_D_aux_upper .* L_B9_LD)));
% Force that is perpendicular to the plane of bearing #3(tangential force),
% absolute value
```

```
        F_r_B9 = sqrt((F_apc_B9).^2 + (F_ppc_B9).^2);
% Radial force applying to bearing #2 (tapered roller bearing)
        F_r_B10 = sqrt((F_apc_B10).^2 + (F_ppc_B10).^2);
% Radial force applying to bearing #3 (tapered roller bearing)




        % Tapered Roller Bearing #7 on counter shaft(front) torque loss
        % JRV this just got a lot harder owing to the possibility of the
        % forces being arrays.
        LF7 = abs(K78910.*F_apc_B7./F_r_B7);
% Counter shaft front bearing(bearing #2) load factor
        [countx,county]=size(LF7);
        f1_B7=zeros(countx,county);
        for counta=1:1:countx
            for countb=1:1:county
                if LF7(counta,countb) < 0.47
                    f1_B7(counta,countb) = 0.06;
                elseif 0.47 <= LF7(counta,countb) < 0.94
                    f1_B7(counta,countb) = -0.13443 - (0.15316 * LF7(counta,countb)) +
(1.238*(LF7(counta,countb)^2));
                elseif 0.94 <= LF7(counta,countb) < 2.0
                    f1_B7(counta,countb) = -0.13538 + (0.96978 * LF7(counta,countb)) +
(0.028991*(LF7(counta,countb)^2)) ;
                else
                    f1_B7(counta,countb) = LF7(counta,countb);
                end
            end
        end
```

```
    torqueloss_B7 = (2.56 .* 10.^-6 .* G78910) .* ((RPM_C_aux .*
mu_oil_dynamic).^0.62) .* ((f1_B7.*F_r_B7./K78910).^0.3);
% Counter shaft front bearing(bearing #2) torque loss



    % Tapered Roller Bearing #8 on counter shaft(Rear) torque loss
    LF8 = abs(K78910.*F_apc_B8./F_r_B8);
% Counter shaft front bearing(bearing #2) load factor
    f1_B8=zeros(countx,county);
    for counta=1:1:countx
      for countb=1:1:county
        if LF8(counta,countb) < 0.47
          f1_B8(counta,countb) = 0.06;
        elseif 0.47 <= LF8(counta,countb) < 0.94
          f1_B8(counta,countb) = -0.13443 - (0.15316 * LF8(counta,countb)) +
(1.238*(LF8(counta,countb)^2));
        elseif 0.94 <= LF8 < 2.0
          f1_B8(counta,countb) = -0.13538 + (0.96978 * LF8(counta,countb)) +
(0.028991*(LF8(counta,countb)^2)) ;
        else
          f1_B8(counta,countb) = LF8(counta,countb);
        end
      end
    end
    torqueloss_B8 = (2.56 .* 10.^-6 .* G78910) .* ((RPM_C_aux .*
mu_oil_dynamic).^0.62) .* ((f1_B8.*F_r_B8./K78910).^0.3);
% Counter shaft front bearing(bearing #2) torque loss
```

```
 % Tapered Roller Bearing #9 on counter shaft(front) torque loss
        LF9 = abs(K78910.*F_apc_B9./F_r_B9);
% Counter shaft front bearing(bearing #2) load factor
        f1_B9=zeros(countx,county);
        for counta=1:1:countx
           for countb=1:1:county
              if LF9(counta,countb) < 0.47
                 f1_B9(counta,countb) = 0.06;
              elseif 0.47 <= LF9(counta,countb) < 0.94
                 f1_B9(counta,countb) = -0.13443 - (0.15316 * LF9(counta,countb)) +
(1.238*(LF9(counta,countb)^2));
              elseif 0.94 <= LF9(counta,countb) < 2.0
                 f1_B9(counta,countb) = -0.13538 + (0.96978 * LF9(counta,countb)) +
(0.028991*(LF9(counta,countb)^2)) ;
              else
                 f1_B9(counta,countb) = LF9(counta,countb);
              end
           end
        end
        torqueloss_B9 = (2.56 .* 10.^-6 .* G78910) .* ((RPM_C_aux .*
mu_oil_dynamic).^0.62) .* ((f1_B9.*F_r_B9./K78910).^0.3);
% Counter shaft front bearing(bearing #2) torque loss


        % Tapered Roller Bearing #10 on counter shaft(Rear) torque loss
        LF10 = abs(K78910.*F_apc_B10./F_r_B10);
% Counter shaft front bearing(bearing #2) load factor
        f1_B10=zeros(countx,county);
```

```
for counta=1:1:countx
    for countb=1:1:county
        if LF10(counta,countb) < 0.47
            f1_B10(counta,countb) = 0.06;
        elseif 0.47 <= LF10(counta,countb) < 0.94
            f1_B10(counta,countb) = -0.13443 - (0.15316 * LF10(counta,countb)) +
(1.238*(LF10(counta,countb)^2));
        elseif 0.94 <= LF10 < 2.0
            f1_B10(counta,countb) = -0.13538 + (0.96978 * LF10(counta,countb)) +
(0.028991*(LF10(counta,countb)^2)) ;
        else
            f1_B10(counta,countb) = LF10(counta,countb);
        end
    end
end
torqueloss_B10 = (2.56 .* 10.^-6 .* G78910) .* ((RPM_C_aux .*
mu_oil_dynamic).^0.62) .* ((f1_B10.*F_r_B10./K78910).^0.3);                    %
Counter shaft front bearing(bearing #2) torque loss


torque_D_L_aux_lower = torque_d_IS_aux_lower - torque_i_d_aux_lower -
torque_L_D_aux_lower - torqueloss_B7 - torqueloss_B8 ;            % Torque delivered
to the 1st drive gear on counter shaft
torque_D_L_aux_upper = torque_d_IS_aux_upper - torque_i_d_aux_upper -
torque_L_D_aux_upper - torqueloss_B9 - torqueloss_B10 ;           % Torque delivered
to the 1st drive gear on counter shaft
```

```
torque_d_L_aux = (torque_D_L_aux_lower .* G_L_aux .* GPEff_L_aux) +
(torque_D_L_aux_upper .* G_L_aux .* GPEff_L_aux);                    % Torque
delivered to the 1st driven gear on output shaft



%%% Force Analysis of driven gear on Aux. stage output shaft
F_t_L_d_aux =  (1000 .* torque_d_L_aux .* R_IO_aux)./(R_p_d_L_aux);
% Absolute tangential force of 1st driven gear
F_s_L_d_aux =  (abs(F_t_L_d_aux).* tand(alpha_n)./cosd(gamma_L_aux));
% Absolute seperating force of 1st driven gear
F_a_L_d_aux =  H_IO_aux .* R_IO_aux .*
abs(F_t_L_d_aux).*tand(gamma_L_aux);
% Absolute axial force of 1st driven gear


F_apc_B11 = abs((1./L_B11_aux) .* ((F_a_L_d_aux .* R_p_d_L_aux) +
(F_s_L_d_aux .* L_d_aux))) ;
% Force that is aligned with the plane of bearing #3(axial force), absolute
% value
F_ppc_B11 = abs((1./L_B11_aux) .* ((F_t_L_d_aux .* L_d_aux))) ;
% Force that is perpendicular to the plane of bearing #2(tangential force),
% absolute value

F_r_B11 = sqrt((F_apc_B11).^2 + (F_ppc_B11).^2);                            %
Radial force applying to bearing #2 (tapered roller bearing)


 % Tapered Roller Bearing #11 on output shaft(Rear) torque loss
```

```
        LF11 = abs(K11.*F_apc_B11./F_r_B11);
% Counter shaft front bearing(bearing #2) load factor
        f1_B11=zeros(countx,county);
        for counta=1:1:countx
            for countb=1:1:county
                if LF11(counta,countb) < 0.47
                    f1_B11(counta,countb) = 0.06;
                elseif 0.47 <= LF11(counta,countb) < 0.94
                    f1_B11(counta,countb) = -0.13443 - (0.15316 * LF11(counta,countb)) +
(1.238*(LF11(counta,countb)^2));
                elseif 0.94 <= LF11(counta,countb) < 2.0
                    f1_B11(counta,countb) = -0.13538 + (0.96978 * LF11(counta,countb)) +
(0.028991*(LF11(counta,countb)^2)) ;
                else
                    f1_B11(counta,countb) = LF11(counta,countb);
                end
            end
        end
        torqueloss_B11 = (2.56 .* 10.^-6 .* G11) .* ((RPM_O_aux .*
mu_oil_dynamic).^0.62) .* ((f1_B11.*F_r_B11./K11).^0.3);
% Counter shaft front bearing(bearing #2) torque loss



        torque_d_L_aux_final = torque_d_L_aux - torque_L_d_aux - torqueloss_B11;
% Final torque of 1st driven gear on output shaft
        torque_d_OS_aux = torque_d_L_aux_final - torqueloss_seal_output;
% Transmission output torque when 1st gear is engaged
```

```
        trans_efficiency_aux = (torque_d_OS_aux./ (input_torque_aux .* G_i_aux .*
G_L_aux));
% Overall Transmission efficiency at 1st gear
        torque_d_OS_aux = torque_d_OS_aux./peak_torque./G_i_aux./G_L_aux;
        %trans_efficiency_final =  trans_efficiency * trans_efficiency_aux;

%        fprintf('Efficiency of high speed at %d rpm is: %5.4f \n\n',
% input_RPM_aux, trans_efficiency_aux)
% Print the efficiency
        %fprintf('Efficiency of %d rpm is: %5.4f \n\n', input_RPM,
% trans_efficiency_final)
% Print the efficiency




    case 2

        RPM_O_aux = RPM_aux;                                        % Output shaft RPM (is
counter shaft RPM divided by 1st gear pair ratio

        torque_oil_L_d_aux = 1.06948 .* 10.^-4 .* rho_oil .* C_E .* (1+ (2.3 .*
(L_FW_L_aux./R_p_d_L_aux))) .* ((RPM_O_aux).^1.85) .* (OD_d_L_aux .* 10.^-
3).^4.7 .* (mu_oil).^0.15;
% Windage torque loss from oil of 1st driven gear
        torque_air_L_d_aux = 1.06948 .* 10.^-4 .* rho_air .* C_E .* (1+ (2.3 .*
(L_FW_L_aux./R_p_d_L_aux))) .* ((RPM_O_aux).^1.85) .* (OD_d_L_aux .* 10.^-
3).^4.7 .* (mu_air).^0.15;
% Windage torque loss from air of 1st driven gear
```

```
torque_L_d_aux = (X_d_L_aux .*torque_oil_L_d_aux) + ((1-X_d_L_aux).*torque_air_L_d_aux);    % Windage torque loss on 1st driven gear



F_t_i_d_aux_lower =  (1000 .* torque_i_d_aux_lower .* R_C_aux)./(R_p_d_i_aux);
% Absolute tangential force of initial drive gear
F_s_i_d_aux_lower =  (abs(F_t_i_d_aux_lower).* tand(alpha_n)./cosd(gamma_i_aux));
% Absolute seperating force of initial drive gear
F_a_i_d_aux_lower =  H_C_aux .* R_C_aux .* abs(F_t_i_d_aux_lower) .* tand(gamma_i_aux);
% Absolute axial force of initial drive gear

F_t_L_D_aux_lower =  (1000 .* torque_L_D_aux_lower .* R_C_aux)./(R_p_D_L_aux);
% Absolute tangential force of 1st drive gear
F_s_L_D_aux_lower =  (abs(F_t_L_D_aux_lower).* tand(alpha_n)./cosd(gamma_L_aux));
% Absolute seperating force of 1st drive gear
F_a_L_D_aux_lower =  H_C_aux .* R_C_aux .* abs(F_t_L_D_aux_lower) .* tand(gamma_L_aux);
% Absolute axial force of 1st drive gear



F_t_i_d_aux_upper =  (1000 .* torque_i_d_aux_upper .* R_C_aux)./(R_p_d_i_aux);
```

```
% Absolute tangential force of initial drive gear
        F_s_i_d_aux_upper =  (abs(F_t_i_d_aux_upper).*
tand(alpha_n)./cosd(gamma_i_aux));
% Absolute seperating force of initial drive gear
        F_a_i_d_aux_upper =  H_C_aux .* R_C_aux .* abs(F_t_i_d_aux_upper) .*
tand(gamma_i_aux);
% Absolute axial force of initial drive gear

        F_t_L_D_aux_upper =  (1000 .* torque_L_D_aux_upper .*
R_C_aux)./(R_p_D_L_aux);
% Absolute tangential force of 1st drive gear
        F_s_L_D_aux_upper =  (abs(F_t_L_D_aux_upper).*
tand(alpha_n)./cosd(gamma_L_aux));
% Absolute seperating force of 1st drive gear
        F_a_L_D_aux_upper =  H_C_aux .* R_C_aux .* abs(F_t_L_D_aux_upper) .*
tand(gamma_L_aux);
% Absolute axial force of 1st drive gear



        %%% Force momentume equations for tapered roller bearing #7 and #8
        F_apc_B7 = abs((-1./L_B7_B8).*((F_s_i_d_aux_lower .* L_d_B8) +
(F_s_L_D_aux_lower .* L_LD_B8) + (F_a_i_d_aux_lower .* R_p_d_i_aux) +
(F_a_L_D_aux_lower .* R_p_D_L_aux))) ;
% Force that is aligned with the plane of bearing #2(axial force), absolute
% value
        F_apc_B8 = abs((1./L_B7_B8) .* ((F_a_i_d_aux_lower .* R_p_d_i_aux) +
(F_a_L_D_aux_lower .* R_p_D_L_aux) - (F_s_i_d_aux_lower .* L_B7_d) -
(F_s_L_D_aux_lower .* L_B7_LD))) ;
```

```
% Force that is aligned with the plane of bearing #3(axial force), absolute
% value
    F_ppc_B7 = abs((1./L_B7_B8) .* ((F_t_i_d_aux_lower .* L_d_B8) +
(F_t_L_D_aux_lower .* L_LD_B8)));
% Force that is perpendicular to the plane of bearing #2(tangential force),
% absolute value
    F_ppc_B8 = abs((1./L_B7_B8) .* ((F_t_i_d_aux_lower .* L_B7_d) +
(F_t_L_D_aux_lower .* L_B7_LD)));
% Force that is perpendicular to the plane of bearing #3(tangential force),
% absolute value

    F_r_B7 = sqrt((F_apc_B7).^2 + (F_ppc_B7).^2);
% Radial force applying to bearing #2 (tapered roller bearing)
    F_r_B8 = sqrt((F_apc_B8).^2 + (F_ppc_B8).^2);
% Radial force applying to bearing #3 (tapered roller bearing)



    %%% Force momentume equations for tapered roller bearing #9 and #10
    F_apc_B9 = abs((1./L_B9_B10).*(-(F_s_i_d_aux_upper .* L_d_B10) -
(F_s_L_D_aux_upper .* L_LD_B10) - (F_a_i_d_aux_upper .* R_p_d_i_aux) -
(F_a_L_D_aux_upper .* R_p_D_L_aux))) ;
% Force that is aligned with the plane of bearing #2(axial force), absolute
% value
    F_apc_B10 = abs((1./L_B9_B10) .* ((F_a_i_d_aux_upper .* R_p_d_i_aux) +
(F_a_L_D_aux_upper .* R_p_D_L_aux) - (F_s_i_d_aux_upper .* L_B9_d) -
(F_s_L_D_aux_upper .* L_B9_LD))) ;
% Force that is aligned with the plane of bearing #3(axial force), absolute
% value
```

```
    F_ppc_B9 = abs((1./L_B9_B10) .* ((F_t_i_d_aux_upper .* L_d_B10) +
(F_t_L_D_aux_upper .* L_LD_B10)));
% Force that is perpendicular to the plane of bearing #2(tangential force),
% absolute value
    F_ppc_B10 = abs((1./L_B9_B10) .* ((F_t_i_d_aux_upper .* L_B9_d) +
(F_t_L_D_aux_upper .* L_B9_LD)));
% Force that is perpendicular to the plane of bearing #3(tangential force),
% absolute value

    F_r_B9 = sqrt((F_apc_B9).^2 + (F_ppc_B9).^2);
% Radial force applying to bearing #2 (tapered roller bearing)
    F_r_B10 = sqrt((F_apc_B10).^2 + (F_ppc_B10).^2);
% Radial force applying to bearing #3 (tapered roller bearing)



    % Tapered Roller Bearing #7 on counter shaft(front) torque loss
    LF7 = abs(K78910.*F_apc_B7./F_r_B7);
% Counter shaft front bearing(bearing #2) load factor
    [countx,county]=size(LF7);
    f1_B7=zeros(countx,county);
    for counta=1:1:countx
      for countb=1:1:county
        if LF7(counta,countb) < 0.47
          f1_B7(counta,countb) = 0.06;
        elseif 0.47 <= LF7(counta,countb) < 0.94
          f1_B7(counta,countb) = -0.13443 - (0.15316 * LF7(counta,countb)) +
(1.238*(LF7(counta,countb)^2));
        elseif 0.94 <= LF7(counta,countb) < 2.0
```

```
          f1_B7(counta,countb) = -0.13538 + (0.96978 * LF7(counta,countb)) +
(0.028991*(LF7(counta,countb)^2)) ;
        else
          f1_B7(counta,countb) = LF7(counta,countb);
        end
      end
    end
    torqueloss_B7 = (2.56 .* 10.^-6 .* G78910) .* ((RPM_C_aux .*
mu_oil_dynamic).^0.62) .* ((f1_B7.*F_r_B7./K78910).^0.3);
% Counter shaft front bearing(bearing #2) torque loss


    % Tapered Roller Bearing #8 on counter shaft(Rear) torque loss
    LF8 = abs(K78910.*F_apc_B8./F_r_B8);
% Counter shaft front bearing(bearing #2) load factor
    f1_B8=zeros(countx,county);
    for counta=1:1:countx
      for countb=1:1:county
        if LF8(counta,countb) < 0.47
          f1_B8(counta,countb) = 0.06;
        elseif 0.47 <= LF8(counta,countb) < 0.94
          f1_B8(counta,countb) = -0.13443 - (0.15316 * LF8(counta,countb)) +
(1.238*(LF8(counta,countb)^2));
        elseif 0.94 <= LF8(counta,countb) < 2.0
          f1_B8(counta,countb) = -0.13538 + (0.96978 * LF8(counta,countb)) +
(0.028991*(LF8(counta,countb)^2)) ;
        else
          f1_B8(counta,countb) = LF8(counta,countb);
        end
```

```
            end
        end
        torqueloss_B8 = (2.56 .* 10.^-6 .* G78910) .* ((RPM_C_aux .*
mu_oil_dynamic).^0.62) .* ((f1_B8.*F_r_B8./K78910).^0.3);
% Counter shaft front bearing(bearing #2) torque loss



         % Tapered Roller Bearing #9 on counter shaft(front) torque loss
        LF9 = abs(K78910.*F_apc_B9./F_r_B9);
% Counter shaft front bearing(bearing #2) load factor
        f1_B9=zeros(countx,county);
        for counta=1:1:countx
            for countb=1:1:county
                if LF9(counta,countb) < 0.47
                    f1_B9(counta,countb) = 0.06;
                elseif 0.47 <= LF9(counta,countb) < 0.94
                    f1_B9(counta,countb) = -0.13443 - (0.15316 * LF9(counta,countb)) +
(1.238*(LF9(counta,countb)^2));
                elseif 0.94 <= LF9(counta,countb) < 2.0
                    f1_B9(counta,countb) = -0.13538 + (0.96978 * LF9(counta,countb)) +
(0.028991*(LF9(counta,countb)^2)) ;
                else
                    f1_B9(counta,countb) = LF9(counta,countb);
                end
            end
        end
        torqueloss_B9 = (2.56 .* 10.^-6 .* G78910) .* ((RPM_C_aux .*
mu_oil_dynamic).^0.62) .* ((f1_B9.*F_r_B9./K78910).^0.3);
```

```
% Counter shaft front bearing(bearing #2) torque loss



    % Tapered Roller Bearing #10 on counter shaft(Rear) torque loss
    LF10 = abs(K78910.*F_apc_B10./F_r_B10);
% Counter shaft front bearing(bearing #2) load factor
    f1_B10=zeros(countx,county);
    for counta=1:1:countx
      for countb=1:1:county
        if LF10(counta,countb) < 0.47
          f1_B10(counta,countb) = 0.06;
        elseif 0.47 <= LF10(counta,countb) < 0.94
          f1_B10(counta,countb) = -0.13443 - (0.15316 * LF10(counta,countb)) +
(1.238*(LF10(counta,countb)^2));
        elseif 0.94 <= LF10(counta,countb) < 2.0
          f1_B10(counta,countb) = -0.13538 + (0.96978 * LF10(counta,countb)) +
(0.028991*(LF10(counta,countb)^2)) ;
        else
          f1_B10(counta,countb) = LF10(counta,countb);
        end
      end
    end
    torqueloss_B10 = (2.56 .* 10.^-6 .* G78910) .* ((RPM_C_aux .*
mu_oil_dynamic).^0.62) .* ((f1_B10.*F_r_B10./K78910).^0.3);
% Counter shaft front bearing(bearing #2) torque loss
```

```
torque_d_L_aux = torque_D_IS_aux - (torque_i_d_aux_upper +
torque_i_d_aux_lower + torque_L_D_aux_upper + torque_L_D_aux_lower) -
(torqueloss_B7 + torqueloss_B8 +  torqueloss_B9 + torqueloss_B10);
% Torque delivered to the 1st driven gear on output shaft



%%% Force Analysis of driven gear on Aux. stage output shaft
F_t_L_d_aux =  (1000 .* torque_d_L_aux .* R_IO_aux)./(R_p_d_L_aux);
% Absolute tangential force of 1st driven gear
F_s_L_d_aux =  (abs(F_t_L_d_aux).* tand(alpha_n)./cosd(gamma_L_aux));
% Absolute seperating force of 1st driven gear
F_a_L_d_aux =  H_IO_aux .* R_IO_aux .*
abs(F_t_L_d_aux).*tand(gamma_L_aux);
% Absolute axial force of 1st driven gear


F_apc_B11 = abs((1./L_B11_aux) .* ((F_a_L_d_aux .* R_p_d_L_aux) +
(F_s_L_d_aux .* L_d_aux))) ;
% Force that is aligned with the plane of bearing #3(axial force), absolute
% value
F_ppc_B11 = abs((1./L_B11_aux) .* ((F_t_L_d_aux .* L_d_aux))) ;
% Force that is perpendicular to the plane of bearing #2(tangential force),
% absolute value

F_r_B11 = sqrt((F_apc_B11).^2 + (F_ppc_B11).^2);                    %
Radial force applying to bearing #2 (tapered roller bearing)
```

```
 % Tapered Roller Bearing #11 on output shaft(Rear) torque loss
    LF11 = abs(K11.*F_apc_B11./F_r_B11);                                                  %
Counter shaft front bearing(bearing #2) load factor
    f1_B11=zeros(countx,county);
    for counta=1:1:countx
      for countb=1:1:county
        if LF11(counta,countb) < 0.47
          f1_B11(counta,countb) = 0.06;
        elseif 0.47 <= LF11(counta,countb) < 0.94
          f1_B11(counta,countb) = -0.13443 - (0.15316 * LF11(counta,countb)) +
(1.238*(LF11(counta,countb)^2));
        elseif 0.94 <= LF11(counta,countb) < 2.0
          f1_B11(counta,countb) = -0.13538 + (0.96978 * LF11(counta,countb)) +
(0.028991*(LF11(counta,countb)^2)) ;
        else
          f1_B11(counta,countb) = LF11(counta,countb);
        end
      end
    end
    torqueloss_B11 = (2.56 .* 10.^-6 .* G11) .* ((RPM_O_aux .*
mu_oil_dynamic).^0.62) .* ((f1_B11.*F_r_B11./K11).^0.3);
% Counter shaft front bearing(bearing #2) torque loss



    torque_d_L_aux_final = torque_d_L_aux - torque_L_d_aux - torqueloss_B11;
% Final torque of 1st driven gear on output shaft
    torque_d_OS_aux = torque_d_L_aux_final - torqueloss_seal_output;
% Transmission output torque when 1st gear is engaged
```

```
        trans_efficiency_aux = (torque_d_OS_aux./ (input_torque_aux));
% Overall Transmission efficiency at 1st gear
        torque_d_OS_aux = torque_d_OS_aux./peak_torque;
        %trans_efficiency_final =  trans_efficiency * trans_efficiency_aux;

%       fprintf('Efficiency of high speed at %d rpm is: %5.4f \n\n',
% input_RPM_aux, trans_efficiency_aux)
% Print the efficiency
        %fprintf('Efficiency of %d rpm is: %5.4f \n\n', input_RPM,
% trans_efficiency_final)
% Print the efficiency
end

% JRV Now to call my program and resolve the quantities
[trans_efficiency_aux,torque_d_OS_aux] =
EtaNTorqCorrect(trans_efficiency_aux,torque_d_OS_aux,true);
[trans_efficiency_aux,torque_d_OS_aux] =
EtaNTorqCorrect(trans_efficiency_aux,torque_d_OS_aux,false);
[diffEff,torque_d_OS_aux,RPM_O_aux]=EffDiffRearHD(torque_d_OS_aux,RPM_O_au
x,diffG);
trans_efficiency_aux=trans_efficiency_aux.*diffEff;


end
```

Section B.6: Efficiency Floor Setting

```
function [ eta,torque ] = EtaNTorqCorrect( eta,torque,zeroornan )
%ETANTORQCORRECT Changes negative efficiencies to zero, corrects torque.
%   Written by James Vaughn
%   Last edited 5 June 2011
%   Inputs:
%   eta - intended for efficiency array.
%   torque - intended for torque fraction array.
%   zeroornan - Make 0 or false to set all NaN elements to zero of both
%               arrays.
%               Make true or any nonzero numeric to set all negative values
%               of efficiency to NaN.

    [a,b]=size(eta);
if zeroornan
    for c=1:1:a
        for d=1:1:b
            if eta(c,d)<0
                eta(c,d)=NaN;
                torque(c,d)=NaN;
            end
        end
    end
else
    e=isnan(eta);
    for c=1:1:a
        for d=1:1:b
            if e(c,d)
```

```
            eta(c,d)=0;
            torque(c,d)=0;
        end
      end
    end
  end

  end
```

Section B.7: Density and Viscosity Models

```
function [rho,mu,nu] = walthersEqn(index)
%walthersEqn Lubricant properties at operating temperature 70 degC
%   Written by James Vaughn
%   Last edited 5 June 2011

T=70+273.15;
% Operating temperature in Kelvin, determined to be 50 Kelvin above normal
% temperatures

% Choosing the correct lubricant from the data available

switch index
    case 1
        lubricantString='ATF.';
    case 2
        lubricantString='75W90.';
    case 3
        lubricantString='80W90.';
    case 0
        lubricantString='Air.';
end

% disp('The specified lubricant is: ');
% disp(lubricantString);

cte=7e-4;      % Coefficient of thermal expansion, degC^-1
```

```
if index==0
   b=1.458e-6;
% kg/(m.s.K^1/2), from The U.S. Standard Atmosphere (1976).
   s=110.4;
% K, from The U.S. Standard Atmosphere (1976).
   mu=b*T^(1/2)/(1+s/T);
% Pa.s, Sutherland's model from The U.S. Standard Atmosphere (1976).
   density=1.2250;
% kg/m^3, from Fox, McDonald, and Pritchard's Introduction to Fluid Dynamics,
% 6th ed.
   rho=density*(1-cte*(T-288.2))^-1;
% Assuming air takes up as much space as the fluid in the enclosure.
   nu=mu/rho;                    % m^2/s.  Defined.
else
   % Obtained from various sources

   density=[854,859,887];       % Density at standard temperature
   v1=[7.2,15.9,15];            % Kinematic viscosity, given in cSt, at T1
   T1=[373.15,373.15,373.15];   % Temperature, in Kelvin
   v2=[35,120,139];             % Kinematic viscosity, given in cSt, at T2
   T2=[313.15,313.15,313.15];   % Temperature, in Kelvin

   % Calculating intermediate values and the final kinematic viscosity


temp=[1,log10(T1(index));1,log10(T2(index))]\[log10(log10(v1(index)+0.8));log10(log1
0(v2(index)+0.8))];
   A=temp(1);
   B=temp(2);
```

```
    nu=10^(10^(A+B*log10(T)))-0.8;                                % Equation 8
from www.eng-tips.com/viewthread.cfm?qid=181824, accessed
% 2/11/2011; kinematic viscosity in centiStoke

    nu=nu*1e-6;              % Converting from cSt to m^2/s

    rho=density(index)*(1+cte*(T-288.2))^-1;

    mu=nu*rho;
end

end
```

Appendix C: Pictures

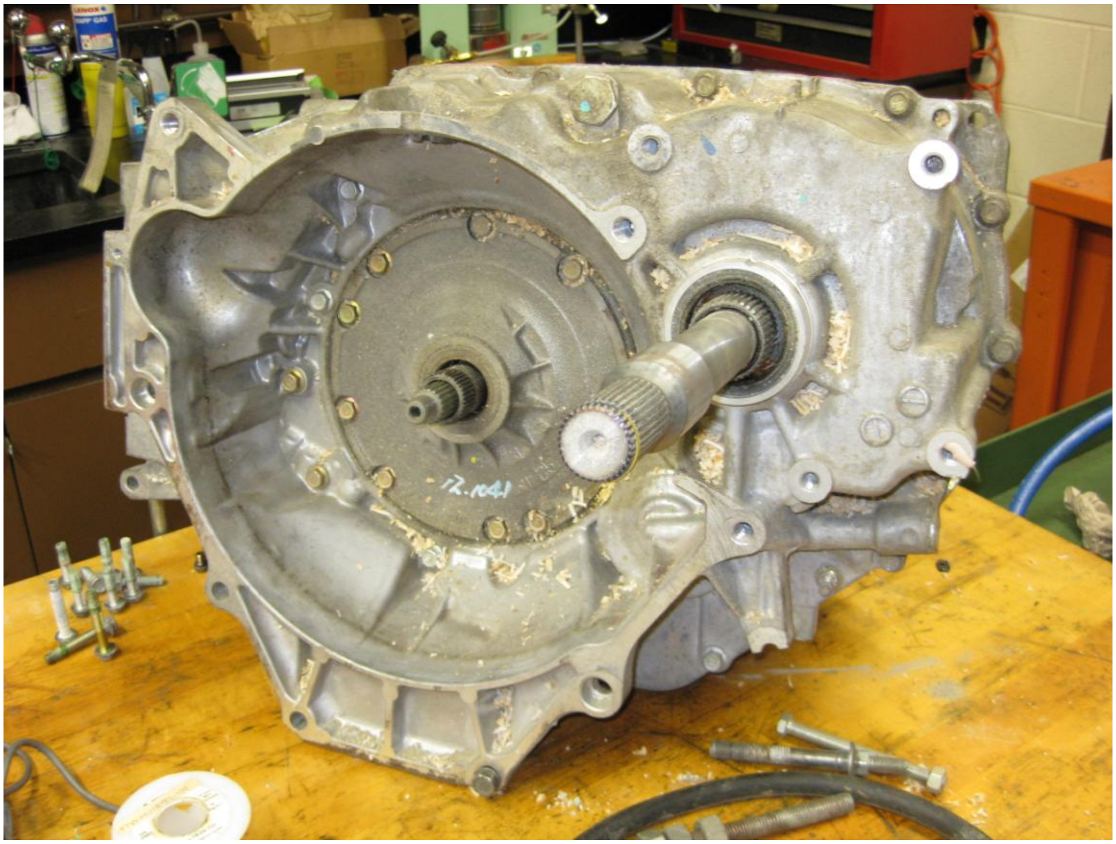


Figure C.1: Transaxle from Chevy Equinox, Upside-Down from Driver's Right Looking Left

Figure C.2: Transaxle from Chevy Equinox, Upside-Down from Driver's Left Looking Right, with a 0.5 Liter Bottle of Automatic Transaxle Fluid Drained from the Transaxle

Figure C.3: Some Complex Transmission Gearing, Bearings, and Clutching Mechanisms from the Transaxle of the Chevy Equinox

Figure C.4: Ring and Pinion Hypoid Gears for a Differential for a Generic 1500-Series Rear-Wheel-Drive Light-Duty Vehicle, at Inland Truck Parts

Figure C.5: Spider Gear Carrier for a Differential for a Generic 1500-Series Rear-Wheel-Drive Light-Duty Vehicle, at Inland Truck Parts, Showing the Surface where the Ring Hypoid Gear Bolts to the Carrier

Figure C.6: Two Spider Gears, a Connecting Rod, Washers, and Female Spline Connectors for a Differential for a Generic 1500-Series Rear-Wheel-Drive Light-Duty Vehicle, at Inland Truck Parts

Figure C.7: Bearings and Ring Hypoid Gear Fasteners for a Differential for a Generic 1500-Series Rear-Wheel-Drive Light-Duty Vehicle, at Inland Truck Parts

Figure C.8: Disassembled Typical Heavy-Duty Conventional Differentials with a Tape Measure for Reference, Tape Measure Tape having 1/4" Width

Figure C.9: Assembled Typical Heavy-Duty Dual Differential Resting Approximately One Foot above Photographer's Size 16 Shoes

Appendix D: Nomenclature

$\alpha =$ Normal pressure angle between gear teeth

$\vec{\alpha}_i =$ Angular acceleration vector projection in direction i

$\alpha_V =$ Volumetric coefficient of thermal expansion for a lubricating liquid

$\gamma =$ Helix angle of a helical or hypoid gear

$\varepsilon =$ Overlap ratio for gears

$\eta =$ Efficiency

$\eta_i =$ Efficiency of component i

$\mu_i =$ Dynamic viscosity of substance i

$\mu_{f,gp} =$ Coefficient of friction of the gear pair

$\nu_i =$ Kinematic viscosity of substance i

$\rho_i =$ Density of substance i

$\rho_{i,j} =$ Density of substance i at temperature j

$\tau =$ Torque

$\tau_i =$ Torque attributed to source i

$\phi_n =$ Normal pressure angle between gear teeth

$\psi_i =$ Helix angle of helical or hypoid gear i

$\omega =$ Angular velocity

$\vec{\omega}_i =$ Angular velocity vector projection in direction i

$\omega_i =$ Angular velocity attributed to source i

$2speed\_gb =$ Pertaining to the two-speed gearbox nestled in a dual differential

$A =$ A constant in Walther's equation or a coastdown coefficient, as defined nearby

$B =$ A constant in Walther's equation or a coastdown coefficient, as defined nearby

$C =$ Center-to-center distance of a gear pair, unless otherwise defined as a constant for Walther's equation or a coastdown coefficient

C_E = Gear enclosure constant, from Townsend's work as cited in Matthews' work

D = Diameter, e.g. of a cylinder, unless pertaining to the driving, or pinion, gear of a gear pair or set

E = Energy

E_i = Energy attributed to source i

F = Force

$\vec{F}_i$ = Force vector projection in direction i

$F'_{\tan}$ = Tangential force per unit circumference of a seal

FW = Face width of a gear tooth

G = Gear ratio

GW = Width of a gear

HD_Dual = Pertaining to the heavy-duty rear wheel drive configuration with a dual differential

HD_RWD = Pertaining to the conventional heavy-duty rear wheel drive configuration

HD_Tan_Conv = Pertaining to the heavy-duty conventional tandem axle configuration

HD_Tan_Dual = Pertaining to the heavy-duty tandem axle configuration with dual differentials

I = Moment of inertia

L = Length, e.g. of a cylinder

L_{fw} = Face width of a gear tooth of a gear pair or set

LD_4WD = Pertaining to the light-duty four wheel drive configuration

LD_AWD = Pertaining to the light-duty all wheel drive configuration

LD_FWD = Pertaining to the light-duty front wheel drive configuration

LD_RWD = Pertaining to the light-duty rear wheel drive configuration

$\vec{M}$ = Moment vector

$\vec{M}_i$ = Moment vector projection in direction i

M_L = Lubricant friction component of the bearing moment

M_P = Load-dependent component of the bearing moment

M_S = Seal contribution to the bearing moment

N = Rotational speed, in rpm

N_i = Rotational speed of gear i, in rpm

OD_i = Outer diameter of gear i

R = Outer radius, e.g. the inner radius of a spherical shell in which a smaller sphere can be found

R_f = Rough surface adjustment factor, from *Dudley's Gear Handbook*

$R_{p,i}$ = Pitch radius of gear i

$R_{i,gr}$ = Inner radius of driven gear of a gear pair or set

S = A constant for the Sutherland model or vehicle speed, in the case of the light-duty rear wheel drive curve fit

T = Temperature, typically in Kelvin

T_0 = Original or reference temperature

W = Work

X = Immersion fraction

a = Clearance, e.g. between a cylinder and a cylindrical shell

$\vec{a}_i$ = Acceleration vector projection in direction i

air = Pertaining to air

b = Pertaining to bearings, unless otherwise defined as a constant of the Sutherland model

$bear$ = Pertaining to bearings

$component$ = Pertaining to quantities internal to or of a theoretical component

cyl = Pertaining to cylindrical geometries

d = Pertaining to the driven gear of a gear pair or set

$diff$ = Pertaining to a differential or configuration of differentials

f = Friction, e.g. of a gear pair

fru = Pertaining to frustal geometries

gb = Pertaining to a gearbox, e.g. of a transfer case

$gear$ = Pertaining to the driven gear of a gear pair or set

gp = Pertaining to a gear pair

gr = Pertaining to the driven gear of a gear pair or set

h_{tooth} = Height of a gear tooth, measured from the inner diameter of a gear to the outer diameter of the gear

hel = Pertaining to a helical gear or gear pair

hyp = Pertaining to a hypoid gear or gear pair

in = Pertaining to a quantity entering the system in question

m = Mass

n_i = Number of teeth on gear i

oil = Pertaining to a petroleum-based or synthetic oil-type lubricant

out = Pertaining to a quantity leaving the system in question

p = Power, unless relating to the pitch diameter or radius of a gear

p_i = Power attributed to source i

pin = Pertaining to the pinion gear of a gear pair or set

$pinion$ = Pertaining to the pinion gear of a gear pair or set

$pitch$ = Pertaining to a pitch radius or pitch diameter

r = Radius, e.g. of a sphere

$r_{p,i}$ = Pitch radius of gear i

s = Pertaining to seals

$seal$ = Pertaining to seals

sph = Pertaining to spherical geometries

tc = Pertaining to a transfer case of a four wheel drive configuration

v = Velocity

$\vec{v}_i$ = Velocity vector projection in direction i

w = Pertaining to windage losses

$wind$ = Pertaining to windage losses

x = Direction arbitrarily chosen to demonstrate relationship of linear kinetics with rotational kinetics

z = Direction arbitrarily chosen to demonstrate relationship of rotational kinetics with linear kinetics

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